Foreign debt supply in an imperfect international capital market: Theory and evidence

Keunsuk Chung\textsuperscript{a}, Stephen J. Turnovsky\textsuperscript{b,}\textsuperscript{*}

\textsuperscript{a}School of Business Administration, Pennsylvania State University, Harrisburg, Middletown, PA 17057, USA
\textsuperscript{b}Department of Economics, University of Washington, Seattle, WA 98195, USA

\textbf{Abstract}

We investigate the determinants of foreign borrowing costs in a stochastically growing economy. We find that these increase with the debt-wealth ratio, depending also upon the volatilities of domestic and foreign origin, and the length of debt contract. In addition, the sensitivity of the short-term debt supply to the debt-wealth ratio exceeds that of long-term debt, and the effects of volatility on the borrowing premium, growth of wealth, and its volatility, depend on the relative size of a direct effect and a secondary portfolio-adjustment effect of the initial shock, as well as the length of the debt contract. Panel regressions suggest that the empirical evidence generally support the theoretical predictions.

\textcopyright{} 2009 Elsevier Ltd. All rights reserved.

1. Introduction

The basic macrodynamic model of a small open economy typically assumes that the country has unrestricted access to a perfect world capital market, so that it can borrow or lend unlimited amounts at an exogenous constant world interest rate. If, in addition, one invokes the conventional assumption of a fixed constant rate of time preference, these two exogenous constants must in fact be equal in order for an interior equilibrium to prevail. This imposes a stringent “knife-edge” condition on the economy, one that has several important ramifications; see e.g. Turnovsky (2002), Schmitt-Grohé and Uribe (2003). For example, it leads to an extreme form of “consumption-smoothing”, and has the further consequence that a temporary structural change or policy shock may have a permanent effect;

\* Corresponding author. Tel.: +1 206 685 8028; fax: +1 206 6857477.
E-mail address: sturn@u.washington.edu (S.J. Turnovsky).
see e.g. Sen and Turnovsky (1989). It also has profound implications for the long-run viability of tax policy; see e.g. Frenkel et al. (1991) and Turnovsky (1997, chapter 6).

Quite apart from the unpalatable practice of imposing equality upon two seemingly unrelated and independent structural parameters, the assumption that a small economy can borrow or lend unlimited amounts indefinitely in international markets at a constant rate is implausible. At some point the economy will cease to be “small” and its decisions will influence the world capital market. Accordingly, beginning with an early paper by Bardhan (1967), economists have periodically imposed a relationship between the rate at which a country can trade financial assets and its net asset position, thereby breaking the knife-edge condition noted above. The effect of this is to introduce a borrowing premium, which essentially serves as a proxy for the country's default risk. Various specifications of this relationship can be found. Several authors follow Bardhan’s original specification and assume that the premium depends upon the level of debt; see e.g. Obstfeld (1982), Bhandari et al. (1990), and Fisher (1995). However, as originally argued by Sachs and Cohen (1982) and Sachs (1984), a more appropriate measure reflecting the country’s ability to service its debt, is to assess the debt relative to some measure of earning capacity, such as its wealth, capital stock or level of output. Moreover, normalizing in this way becomes necessary if one wishes to incorporate increasing debt costs in an equilibrium of ongoing growth; see e.g. van der Ploeg (1996), Turnovsky (1997), and Turnovsky and Chattopadhyay (2003); see also Mendoza and Uribe (2000), and Sendhadji (2003).

Underlying these specifications is the potential for default risk of highly indebted economies. The issue of sovereign default risk and debt repayment stems from the seminal work of Eaton and Gersovitz (1981, 1989). Emphasizing the difference between sovereign default and bankruptcy of an individual agent, they present a debt repayment function in which lenders establish a credit ceiling that prevents borrowers from repudiating the debt. Similarly, many researchers focus on the sovereign government’s trade off between default and lender-enforced penalties, such as a ban from international credit market (Grossman and van Huyck, 1988), seized assets abroad, or future trade barriers (Bulow and Rogoff, 1989).

The upward-sloping supply of debt function has been employed in a variety of contexts. These include: an analysis of the terms of trade shocks (Obstfeld, 1982; Eicher et al., 2008), international term structure of interest rates (Fisher, 1995), economic growth (van der Ploeg, 1996; Turnovsky and Chattopadhyay, 2003) and foreign aid (Chatterjee et al., 2003). The idea of inelastic supply curve is also applied in the models of financial crisis episodes in less developed countries. Duffie et al. (2003) model the pricing of sovereign debt focusing on the yield spread. Eicher et al. (2001) show that financial market liberalization alone can generate sharp reversals in foreign capital flows if short-term debt supply is more inelastic than is long-term debt. Rodrik and Velasco (2000) provide a theoretical and empirical analysis of the effects of socially excessive short-term capital flows as a cause of more severe crisis.

While the specification of borrowing costs as an increasing function of the country’s net foreign debt position seems plausible, it is nevertheless ad hoc. Beginning with Bardhan (1967) it has been postulated as an equilibrium reduced form relationship, rather than being derived from the underlying behavior of relevant agents. As a consequence, the existing literature simply postulates an arbitrary (usually convex) relationship between debt and borrowing costs, ignoring the dependence of this relationship upon other relevant aspects, such as the degree and nature of the underlying risk in the economy.

This paper has several objectives. First, we wish to derive the equilibrium borrowing premium from the rational behavior of risk-neutral expected profit maximizing international financial intermediaries that allocate funds between borrowers and lenders, while taking account of the potential for default risk. Domestic borrowers are subject to two sources of risk, (i) an internal source, due to domestic production risk, and (ii) an external source, due to the stochastic pricing of foreign bonds, as a result of which they risk defaulting on their loans. We show how this leads to an equilibrium in which the borrowing cost increases with the economy’s debt-wealth ratio, with the relationship depending upon the domestic/foreign sources of risk and the length of debt contract.

---

1 There are other ways to break the knife-edge condition, one of which is to endogenize the rate of time preference, by the use of Uzawa (1968) preferences for example. However, as several authors have noted, this requires unsatisfactory restrictions in order for well-behaved dynamics to obtain; see e.g. Blanchard and Fisher (1989).

Please cite this article in press as: Chung, Keunsuk, Stephen J. Turnovsky, Foreign debt supply in an imperfect international capital market: Theory and evidence, Journal of International Money and Finance (2009), doi:10.1016/j.jimonfin.2009.06.015
This equilibrium relationship turns out to be a complex one and we therefore analyze it numerically, by calibrating a plausible replication of a representative small open debtor economy under various conditions of risk and borrowing costs. Any changes in these factors will shift the supply curve of debt, causing subsequent reallocation of resources. Along the supply schedule, the agent’s resource reallocation further adjusts the borrowing cost. We also find that the curvature of the debt supply curve, a critical determinant of the borrowing premium, is highly sensitive to the length of the debt contract. Although the short-term debt supply curve begins at a lower interest rate with flatter slope than does the one with a longer maturity, its shape dramatically changes with the increase in the relative size of debt. This result is consistent with the findings in the debt crisis literature (for example, Rodrik and Velasco, 2000).

After developing the debt cost function in detail, we then introduce it into the stochastic dynamic general equilibrium growth model of Turnovsky and Chattopadhyay (2003). The resulting equilibrium is a balanced growth path along which all variables grow at a common stochastic rate, determined by the parameters of preference, production, and risks. In doing so, we focus on comparing the consequences of replacing the arbitrarily specified debt-cost function, employed by Turnovsky and Chattopadhyay with the form derived in this paper. We find that the simplified specification of the debt-cost function generally leads to an understatement of the borrowing premium in the case of short-term debt and overstatement in the case of long-term debt.

The final stage of our analysis is to supplement our theoretical and numerical analysis with empirical evidence. Panel regressions of 20 developing debtor economies over 42 quarters between 1993 and 2004 provide an empirical support to our findings. In particular, we find that the effect of debt on the borrowing cost is steeper for short-term debt, consistent with our numerical simulations. We also find that the borrowing premium for both types of debt increases with the terms of trade volatility, while output volatility has a strong impact on the borrowing premium for long-term debt, but only a marginal effect on the premium for short-term debt. This empirical evidence confirms the importance of deriving the cost of debt function from underlying rational behavior.

The rest of the paper is organized as follows. Section 2 develops a formal analytical model of the interest premium for risky foreign debt in the context of a dynamic general equilibrium. Section 3 presents results from the calibration to analyze the effects of volatility on the balanced growth path equilibrium. In Section 4, we briefly explain the theoretical background of our regressions and develop a basic regression model, followed by the description of the data and regression results. Section 5 summarizes our main conclusions and an Appendix provides some of the technical details.

2. The model

2.1. The economy

The world economy comprises two countries, a small open home economy and a large open foreign country, both of which produce the same traded good, which serves as numeraire.

A key element of the model is the international financial market. Participating economies are exposed to two sources of risk, one of domestic origin, and the other of foreign origin. The small domestic economy, which is our focus, is a developing economy entering the world financial market as a borrower to finance its growth and development. Having insufficient endowment of productive capital, it may be unable to finance its obligations to lenders in the financial market. However, the financial intermediaries do not have complete information on the risks associated with the domestic economy and hence seek to charge an interest rate that can compensate them for potential losses from unforeseen adverse future events. On the other hand, individual domestic borrowers consider themselves so small that they have an unlimited access to the international credit market, at the borrowing rate determined by the financial intermediary.

2.2. Technology and factor payments

We assume that there are $M$ domestic firms. Each representative firm produces the traded output in accordance with the stochastic Cobb–Douglas production function.
\[\text{d}Y_i = a \left( L'K \right)^\eta \left( K^i \right)^{1-\eta} \left( \text{d}t + \text{d}y \right) \equiv f(K) \left( \text{d}t + \text{d}y \right) \quad i = 1, \ldots, M \tag{1a}\]

where \(K^i\) denotes the individual firm’s capital stock, \(L^i\) denotes the individual firm’s employment of labor, \(K\) is the average stock of capital in the economy, so that \(L'K\) measures the efficiency units of labor employed by the firm; see e.g. Corsetti (1997). The stochastic variable is temporally independent, with mean zero and variance \(\sigma_t^2 dt\) over the instant \(dt\). The stochastic production function exhibits constant returns to scale in the private factors – labor and the private capital stock.

All firms face identical production conditions and are subject to the same realization of an economy-wide stochastic shock. Hence they will all choose the same level of employment and capital stock. That is, \(K^i = K\) and \(L^i = L\) for all \(i\), where \(L\) the average economy-wide level of employment, assumed to be fixed. The average capital stock yields an externality such that in equilibrium the aggregate (average) production function is linear in the aggregate capital stock, as in Romer (1986), namely

\[dY = aL^0K(\text{d}t + \text{d}u) \equiv f(K) \left( \text{d}t + \text{d}y \right) \tag{1b}\]

where \(f = aL^0\) and \(\partial f / \partial L > 0\).

We assume that the wage rate, \(a\), over the period \([t, t + \text{d}t]\) is determined at the start of the period and is set equal to the expected marginal physical product of labor over that period. The total rate of return to labor over the same interval is thus specified nonstochastically by

\[dA = a \text{d}t = \left( \frac{\partial f}{\partial L} \right)_{K = K, L = L} \text{d}t \tag{2a}\]

where \(a = \eta K = \eta aL^0K \equiv wK\).

Returns from domestic investment are determined as residuals of the final output after the payments for labor and depreciation are made.

\[dR = \frac{dY - L \text{d}A - \delta \text{d}t}{K} \equiv r \text{d}t + \text{d}u_k \tag{2b}\]

where \(r \equiv \left( \frac{\partial f}{\partial L} \right)_{K = K, L = L} = f(1 - \eta) - \delta\), and \(\text{d}u_k \equiv f \text{d}y\).

These two equations assume that the wage rate, \(a\), is fixed over the time period \([t, t + \text{d}t]\), so that the return to capital absorbs all output fluctuations. Given that firms are identical, we find that the equilibrium return to capital is independent of the stock of capital while the wage rate is proportional to the average stock of capital, and therefore grows with the economy.

The relative price of international bonds \(P\), expressed in terms of the traded good, follows the geometric Brownian motion process:

\[\frac{\text{d}P}{P} = \varepsilon \text{d}t + \text{d}p, \tag{3}\]

where \(\text{d}p \sim N(0, \sigma_t^2 \text{d}t)\) so that the value of the foreign debt, expressed in terms of the numeraire, changes stochastically over time. The default risk-free interest rate charged for foreign loans is \(r^d \text{d}t\) which expressed in terms of the traded good implies the stochastic borrowing rate:\(^2\)

\[\text{d}R^* = (r^d + \varepsilon) \text{d}t + \text{d}p \tag{4}\]

However, domestic borrowers may fail to repay the debt in full, depending on the domestic production and the available capital stock. This possibility of repayment failure causes the international lenders to charge the domestic borrowers a risk premium. The interest rate on the domestic borrowers’ debt then follows analogous stochastic process:

\(^2\) We assume that the stochastic component of the interest rate on international debt is determined only by the risk of the relative price of international borrowing, which for simplicity is taken to be uncorrelated with domestic production risk.
\[ dR_z = (r_z + \varepsilon)dt + dU_z = (r_z + \varepsilon)dt + dp \]

where \( r_z \) includes a risk premium such that \( r_z \geq r^* \). Our next task is to determine this premium.

### 2.3. International financial market

Domestic agents have access to an international financial market. In that market, there are three types of agents – borrowers, savers, and risk-neutral, foreign-owned, financial intermediaries. The financial intermediaries take deposits from savers and lend the proceeds to borrowers for a given contract period. Foreign agents are assumed to be free of default-risk and therefore have unlimited access to international loans at the given riskless world interest rate \( r^* \). Individual domestic borrowers’ access to the international financial market takes place via the domestic central bank, which enters the international loan market on their collective behalf. In this way, lenders assess domestic borrowers’ ability to repay the debt in terms of the aggregate debt position of the domestic economy. Since our primary focus is on the debt supply curve faced by a small open developing economy, we shall limit our discussions to the case where the home country is a debtor and the foreign nation is a creditor.

Once a debt contract is made, a borrower continuously makes interest payments over the contract period. On its maturity date, he makes the last interest payment, repays the principal, and signs a new contract. Since the risk-free borrowing rate (or deposit rate) is given, a representative financial intermediary will set the interest rate on the risky debt so as to maximize the present value of the expected profit from lending the deposits of savers to risky borrowers.

Let \( Z \) denote the number of units of risky foreign debt. This equals \( PZ \) units of traded output, where \( P \) is the relative price of foreign bonds specified by the stochastic process (3). Although the initial loan, \( Z \), is fixed by the contract, its current value fluctuates over time as \( P \) evolves stochastically over time. Suppose that at time \( t \), the domestic agent takes on an initial loan of \( (PZ)_t \), which matures in \( T \) years. At any time \( t + s (0 \leq s \leq T) \), the borrower owes interest equal to \( r_z (PZ)_{t+s} \), and the principal \( (PZ)_{t+T} \) at its expiration date. However, due to the risk of default, what the domestic borrower actually pays may, or may not, equal what he owes. Therefore, if the relative size of debt in the borrower’s wealth grows too large, the financial intermediary takes it as a signal of potential default and re-assesses the expected value of payoff. Let \( (PZ)_{t+s} \) denote the basis of the payoff assessment at time \( t + s \), used by the financial intermediary. Depending on the borrower’s available resources, either \( (PZ)_{t+s} = (PZ)_{t+T} \) or \( (PZ)_{t+s} < (PZ)_{t+T} \) will hold. We can write the expected present value of the payoff to the intermediary from risky debt to be:

\[
E_t^z (\text{Payoff}) = E_t^z \left[ \int_0^T r_z (PZ)_{t+s} e^{-\rho(s-t)} ds + (PZ)_{t+T} e^{-\rho T} \right],
\]

where \( \rho (= r^* + \varepsilon) \) is a risk-free discount rate and \( E_t^z (\cdot) \) is the expected value at time \( t \) based on a risk-neutral probability measure.\(^5\)

We assume that when the borrower’s debt-wealth ratio exceeds a critical level, \( \theta \), the financial intermediary anticipates default on the loan, either of the interest due or of the repayment (if at time \( T \)). Under these circumstances the lender and borrower negotiate that the borrower will compensate the lender by forfeiting a fraction of domestic capital, \( \lambda K_{t+s} \), which is smaller than the actual value of debt, \( (PZ)_{t+s} \).

Define \( A_{t+s} \) as the normal situation where the relative debt size is below the critical level of debt, and \( A_{t+s}^C \) as the situation where it exceeds the critical level. Denoting wealth by \( W \), the condition for \( A_{t+s} \) can be written as\(^6\):

\[ W_t = K_t - (PZ)_t. \]

---

\(^3\) More detailed discussion is provided in Section 2.3 and in the Appendix.

\(^4\) We may think of this situation as though the borrower loans out the principal of expiring debt via a new borrowing contract and rolls it over to the next period.

\(^5\) Some brief informal comments on the concept of risk neutral probability are provided in the Appendix. For more formal discussions of the concept and its application to asset pricing see e.g. Merton (1973), Cox and Ross (1976), Harrison and Kreps (1979) and Harrison and Pliska (1981).

\(^6\) As we will see later, domestic wealth, \( W_t = K_t - (PZ)_t \).
(W_{t+s}, (PZ)_{t+s})|\theta W_{t+s} \geq (PZ)_{t+s}\).

Once the above condition is violated, a value-adjustment is made, but when the debt size returns to normal, the financial intermediary resumes its original evaluation, that is, full repayment. We can rewrite the basis of repayment \((PZ)_{t+s}\) as \((PZ)_{t+s} I_{\Lambda_{t+s}} + \lambda K_{t+s}(1 - I_{\Lambda_{t+s}})\), where \(I_{\Lambda_{t+s}} = 1\) if \(A_{t+s}\) occurs, and 0 otherwise. Then the expected present value of payoff becomes

\[
E_t^r(\text{Payoff}) = \int_0^T r_s e^{-\rho T} E_t^r \left[ \left( \frac{PZ}_{t+s} \right)_{t+s} \right] ds + e^{-\rho T} E_t^r \left[ \left( \frac{PZ}_{t+s} \right)_{t+s} \right] \\
= \int_0^T \left\{ r_s e^{-\rho T} E_t^r \left[ \left( PZ \right)_{t+s} I_{\Lambda_{t+s}} + \lambda K_{t+s} \right(1 - I_{\Lambda_{t+s}}) \right] \right\} ds + e^{-\rho T} E_t^r \left[ \left( PZ \right)_{t+s} I_{\Lambda_{t+s}} \right. \\
+ \lambda K_{t+s} \left(1 - I_{\Lambda_{t+s}} \right) \right] \right\} \\
\]

(6)

Evaluating this expression yields\(^7\)

\[
E_t^r(\text{Payoff}) = r_s \int_0^T \left\{ (PZ)_t \Phi(h_1) e^{-r^s} + \lambda K_t \Phi(h_2) \right\} ds + \left( (PZ)_t \Phi(h_1^T) e^{-r^T} + \lambda K_t \Phi(h_2^T) \right)
\]

(7)

where

\[
h_1 = - \frac{\bar{n}_k \sqrt{\left( f^2 \sigma^2 + \sigma^2_{p} \right) s}}{2} - \frac{r^s}{\bar{n}_k \sqrt{\left( f^2 \sigma^2 + \sigma^2_{p} \right) s}} - \ln \left[ \frac{\theta_1 \Phi(h_1)}{W_t} \right],
\]

\[
h_2 = - \frac{\bar{n}_k \sqrt{\left( f^2 \sigma^2 + \sigma^2_{p} \right) s}}{2} + \frac{r^s}{\bar{n}_k \sqrt{\left( f^2 \sigma^2 + \sigma^2_{p} \right) s}} + \ln \left[ \frac{\theta_1 \Phi(h_2)}{W_t} \right].
\]

\(\bar{n}_k\) is the capital-wealth ratio on the balanced growth path equilibrium, \(h_i^T\) is \(h_i\) at time \(t + T\) \((i = 1, 2)\), and \(\Phi(\bullet)\) is the cumulative distribution function of a standard normal random variable.

The expected present value in equation (7) determines the financial intermediary's revenue, while its expected present value of payments to the depositors determines its cost. Suppose funds equal to \((PZ)_t\) are financed by deposits from foreign savers. Since the financial intermediary is a default-risk free borrower to the depositors, the expected present value in the cost side is riskless and equal to the value of the initial loan. While the risk-neutral financial intermediary will set the interest rate on the risky debt, \(r_s\), to maximize its expected profit, \(E_t^r(\text{Payoff}) - (PZ)_t\), competition in the financial market will drive the expected profit down to zero. Therefore, in equilibrium, the expected present value of the payoff from the risky borrower and that to the foreign depositor must be equal. With the discount rate \(\rho = r^s + \epsilon\), the zero-profit condition further simplifies to

\[
r_s \int_0^T \left\{ (PZ)_t \Phi(h_1) e^{-r^s} + \lambda K_t \Phi(h_2) \right\} ds + \left( (PZ)_t \Phi(h_1^T) e^{-r^T} + \lambda K_t \Phi(h_2^T) \right) = (PZ)_t
\]

(8)

The right-hand side of equation (8) is the expected present value of the intermediary's total future payments to the foreign depositor, and the left-hand side is the expected value of the possible total future receipts from the risky loans, weighted by the probability of full payment, \(\Phi(h_1)\), and the probability of partial recovery, \(\Phi(h_2)\).

---

\(^7\) Details of the derivation are provided in the Appendix.
Rearranging the terms, we may write (8) as:

\[
  r_z = \frac{1 - \left[ \Phi(h_1^2) e^{-rT} + \frac{\lambda}{(PZ)_t} \Phi(h_2^2) \right]}{\int_0^T \left[ \Phi(h_1) e^{-rs} + \frac{\lambda}{(PZ)_t} \Phi(h_2) \right] ds}
\]

(9)

Using the definition of wealth \( W_t = K_t - (PZ)_t \), we rewrite the expression for the interest rate set by the intermediary as follows:

\[
  r_z = \frac{1 - \left[ \Phi(h_1^2) e^{-rT} + \lambda \left(1 + \frac{W_s}{(PZ)_t}\right) \Phi(h_2^2) \right]}{\int_0^T \left[ \Phi(h_1) e^{-rs} + \lambda \left(1 + \frac{W_s}{(PZ)_t}\right) \Phi(h_2) \right] ds}
\]

(10)

where \( h_1 \) and \( h_2 \) are defined in equation (7).

Equation (10) yields a version of a positively sloped supply curve of foreign debt, analogous to the relationship \( r_z = r^* + \omega((PZ)/W) \) postulated by Turnovsky and Chattopadhyay (2003). In contrast to this simplified relationship where \( \omega(\cdot) \) is some fixed convex function, (10) reveals that the equilibrium borrowing premium depends not only upon the debt-wealth ratio, but also upon the length of debt contract, \( T \), as well as the underlying risk, \( \sigma_z^2, \sigma_p^2 \), as reflected in \( h_1, h_2 \). Moreover, these factors influence one another interactively.

A change in any of these factors will induce subsequent changes in the probabilities of full repayment and partial recovery, and consequently a change in the interest premium. For instance, consider a change in the relative size of debt. Notice that \( h_1 \) and \( h_2 \) are equi-distant from \(-h_k \sqrt{(f^2 \sigma_z^2 + \sigma_p^2)s/2} \), but at opposite ends of the probability distribution. Given the parameter values and maturity, a lower debt-wealth ratio at time \( t \) pushes \( h_1 \) up and \( h_2 \) down, farther away from each other. As the gap between them becomes larger, the probability of full repayment approaches 1 and the probability of partial recovery approaches 0. Therefore, as the relative size of debt decreases, the expected actual repayment at time \( t + s \) becomes closer to what he owes and the need for risk compensation will be less. In the limit, as \( Z \to 0, h_1 \to \infty, h_2 \to -\infty \), implying \( \Phi(h_1) \to 1, \Phi(h_2) \to 0 \), in which case \( r_z \to r^* \).

In case of an increasing debt, the required compensation in the form of interest premium will become higher. Given the debt-wealth ratio, any change in the risk parameters or the contract period also affects the risk premium and the mean interest rate. One thing we should note is that changes in risk shift the entire supply schedule of foreign debt, while changes in the relative debt size moves the economy along the same curve.

Fig. 1 illustrates the surface of the mean interest rate \((r_z + \epsilon)\) for varying maturity and relative debt size for the parameter set summarized in Table 1. In this figure, the debt-wealth ratio ranges between 0 and 0.3, while the maturity ranges from 0 to 15 years. At any given maturity, the debt supply curve is an increasing function of the size of debt, and at any given size of debt, the interest rate shows upward sloping, hump-shaped, or downward sloping, yield curve properties depending on the size of debt.

In Fig. 2, we compare the shapes of short/medium/long-term debt supply curves, which are obtained by drawing planes of Fig. 1. We have chosen maturity lengths of 1, 5, and 10 years, and they are identified by solid, long-dashed, short-dashed lines, respectively. Fig. 2 illustrates that all three curves with different maturities show a convex shape. A debt supply curve with a shorter maturity has a larger curvature. When the amount of debt is small, a shorter-term premium is less sensitive to the debt size than a longer-term premium. However, as the debt size increases, a shorter-term premium goes up faster than a longer-term one. In contrast, the risk premium for the long-term debt increases relatively faster with small size of debt, while the speed of increase slows down as its size grows further. Regarding the length of debt contract, \( T \), there are two types of uncertainty involved. One is related to the borrower’s ability to accumulate sufficient stock of capital by the maturity date, and the other is

---

8 The same applies as the risk terms \( \sigma_z^2, \sigma_p^2 \to 0 \).

9 Illustrating the graph, we approximate the denominator in equation (10) as a discrete sum of 12 subintervals over \( T \) years, assuming monthly interest payment. Increasing the number of discrete payments adds accuracy and more smoothness in the surface at the cost of longer time for computation, without altering the implications of our analysis.
related to the borrower’s credibility to make the interest payments throughout the entire contract period. Given the initial amount of debt, if the contract expires too soon, the borrower may not be able to accumulate sufficient stock of capital for principal payment. On the other hand, if the contract is too long, the lender may not know what will happen in the later period of the contract. For a short-term debt, the latter risk is dominated by the former. Therefore, in the relatively low range of short-term debt, an additional borrowing would not increase the overall risk much, leaving the supply curve relatively more elastic. As the debt size continues to grow, the former risk dramatically increases and the supply curve becomes increasingly inelastic. With a long-term debt, on the contrary, the latter risk is relatively more important. Therefore, an additional long-term borrowing will add more uncertainty about the future repayment through the second channel of risk, but the former risk will not be as dramatic as in the short-term debt case, implying more inelasticity in the low range and less inelasticity later on. Overall, an increase in the relative debt size plays a more significant role later on in the short-term borrowing case, suggesting a more convex shape of supply curve than for long-term debt.

In Fig. 3, we illustrate the effects of two sources of volatility on the position of the debt supply curve. Increasing volatility, whether it is domestic or external, puts an upward pressure on the borrowing rate, hence shifting the debt supply curve to the left. Given the maturity, a change in the external volatility shifts the debt supply curve further than a change in the domestic output volatility. Given the type of volatility, the longer the debt contract, the farther the supply curve shifts.

### Table 1
Benchmark parameter values.

<table>
<thead>
<tr>
<th>Preference parameters</th>
<th>$\beta = 0.03; \gamma = -1.5$;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production parameters</td>
<td>$f = 0.3; \eta = 0.5$;</td>
</tr>
<tr>
<td>Returns, discount rate</td>
<td>$r^* = 0.05; \epsilon = 0.01; \delta = 0.04$;</td>
</tr>
<tr>
<td>Volatility parameters</td>
<td>$\sigma_y = 0.05, 0.10, 0.20$;</td>
</tr>
<tr>
<td></td>
<td>$\sigma_p = 0.08, 0.16, 0.24$;</td>
</tr>
<tr>
<td>Financial market</td>
<td>$\theta = 0.225; \lambda = 0.1$;</td>
</tr>
<tr>
<td>Maturities</td>
<td>$T = 1, 5, 10$;</td>
</tr>
</tbody>
</table>

Note: 1. $\theta$ is set at 0.225. It means that the international financial intermediary starts anticipating a partial recovery when the debt-output ratio or debt-GDP ratio is about 61.2%. Most of the crisis-struck economies in our sample data show about 60–80 percent of debt-GDP ratios during the crises. 2. $\lambda = 0.1$ means that the financial intermediary anticipates to recover 10% of domestic capital as a partial repayment in the worst case. 3. We have three volatility levels—low, medium and high. In all levels, $\sigma_p$ is set higher than $\sigma_y$, reflecting that the variation in real exchange rate (or TOT) tends to be larger than that in domestic output. 4. Time preference parameter $\beta$ is set at 0.03. It only influences the mean growth of wealth and consumption-wealth ratio without affecting the portfolio decision in our model. 5. We consider three different terms to maturity: short, medium and long term. We define a short-term debt as the one expiring in a year, a medium-term in five years, and a long-term in ten years.
3. Macroeconomic equilibrium

We now incorporate the debt supply function (10) into the small stochastic general equilibrium model developed by Turnovsky and Chattopadhyay (2003). In that model, the representative consumer maximizes expected lifetime utility, assumed to be a function of the agent’s consumption, \( C_i(t) \), represented by the isoelastic utility function.  

\[
\max E_0 \int_0^\infty \frac{1}{\gamma} \left( C_i(t) \right)^{\gamma} e^{-\beta t} dt, \quad -\infty < \gamma < 1 
\]  

(11)

where \( 1 - \gamma \) equals the coefficient of relative risk aversion. Empirical evidence suggests that this is relatively large, certainly well in excess of unity, so that we shall assume \( \gamma < 0 \). This maximization is subject to the agent’s capital accumulation constraint

\[
dW_i = K_i^\prime dR_k - PZ_i^\prime dR_z + L^\prime dA - C_i^\prime dt = \left[ r_k K_i^\prime - r_z PZ_i^\prime + wL_i^\prime K - C_i^\prime \right] dt + K_i^\prime du_k - PZ_i^\prime du_z 
\]  

(12a)

where wealth of agent \( i \) is

\[
W_i = K_i - PZ_i 
\]  

(12b)

and \( du_k = f\ dy, du_z = dp \). Wealth is accumulated from the return to investment, less borrowing costs, plus labor income, less consumption.

It is important to observe that with the equilibrium wage rate being tied to the economy’s aggregate stock of capital (see (2a)), the rate of accumulation of the individual’s capital stock depends on the aggregate level of wealth, which, aggregating (12a) over the agents in the economy, evolves in accordance with

\[
dW = (r_k K - (r_z + \varepsilon) PZ + wL K - C) dt + K du_k - PZ du_z 
\]  

(12c)

---

Fig. 2. Foreign debt supply curves: short-, medium- and long-term debt. Note: 1. \( f = 0.3, \sigma_y = 0.1, \sigma_p = 0.16, \theta = 0.225, \lambda = 0.1 \). 2. The shorter the contract period, the larger the curvature of debt-supply curve is. Therefore, the short-term supply curve is flatter in the beginning and becomes steeper as the relative size of debt increases.

---

Note that the constant elasticity function implies that coefficient of relative risk aversion, \( R \), and the intertemporal elasticity of substitution, \( s \), are linked by the constraint \( R = (1 - \gamma) = 1/s \).

The consensus estimates suggest that \( R \) lies in the range 2–5.
Fig. 3-1. Effects of a change in domestic output volatility. a. $\sigma_y$ and the short-term debt supply curve. b. $\sigma_y$ and the long-term debt supply curve. Note: 1. $f = 0.3$, $\theta = 0.225$, $\lambda = 0.1$, $T = 1$ (ST) and 10 (LT). Each panel corresponds to low, medium and high domestic risk (from left to right). 3. Solid lines, short-dashed lines and long-dashed lines represent low, medium and high domestic risk, respectively.

Fig. 3-2: Effects of a change in external volatility. a. $\sigma_p$ and the short-term debt supply curve. b. $\sigma_p$ and the long-term debt supply curve. Note: 1. $f = 0.3$, $\theta = 0.225$, $\lambda = 0.1$, $T = 1$ (ST) and 10 (LT). Each panel corresponds to low, medium and high domestic risk (from left to right). 3. Solid lines, short-dashed lines and long-dashed lines represent low, medium and high external risk, respectively.

Please cite this article in press as: Chung, Keunsuk., Stephen J. Turnovsky,, Foreign debt supply in an imperfect international capital market: Theory and evidence, Journal of International Money and Finance (2009), doi:10.1016/j.jimonfin.2009.06.015
where $L$ denotes the average (aggregate) labor supply. The agent therefore also needs to take this relationship into account in performing his optimization.

To solve the agent’s optimization it is convenient to introduce the portfolio shares

$$n^i_k = \frac{K^i}{K^i - PZ^i}; \quad n^i_z = \frac{PZ^i}{K^i - PZ^i}$$

Written in this way, the consumer’s formal optimization problem is to choose $C^i/W^i$ and portfolio shares to maximize (11) subject to

$$dW^i = \left( r_k n^i_k - (r_z + \varepsilon)n^i_z + \frac{L^i}{W^i} - \frac{C^i}{W^i} \right) dt + \left( n^i_k du_k - n^i_z du_z \right).$$

$$1 = n^i_k - n^i_z$$

$$dW = \left( r_k n_k - (r_z + \varepsilon)n_z + \frac{L}{W} - \frac{C}{W} \right) dt + \left( n_k du_k - n_z du_z \right)$$

where $(12b')$ is obtained from $(12b)$ and $n_k, n_z$ are the average economy-wide portfolio shares prevailing in equilibrium. Since all agents are identical, $W^i = W, n^i_k = n_k, n^i_z = n_z$ hold in equilibrium.

This problem is identical in structure to that formulated by Turnovsky and Chattopadhyay (2003). The only difference is in the form of the borrowing cost which is given by the complex relationship in (10). But since this involves the economy’s aggregate debt position, the agent takes this as given. Following the procedure adopted by Turnovsky and Chattopadhyay, the macroeconomic equilibrium can be summarized by the following relationships 12:

$$\hat{n}_k = \frac{r_k - (r_z + \varepsilon)}{(1 - \gamma)\left(f^2\sigma^2_y + \sigma^2_p\right)} + \frac{\sigma^2_p}{f^2\sigma^2_y + \sigma^2_p}$$

$$\hat{n}_z = \frac{r_k - (r_z + \varepsilon)}{(1 - \gamma)\left(f^2\sigma^2_y + \sigma^2_p\right)} - \frac{f^2\sigma^2_p}{f^2\sigma^2_y + \sigma^2_p}$$

$$\hat{\psi} = \frac{1}{1 - \gamma}\left[ \left( r_k \hat{n}_k - (r_z + \varepsilon) \hat{n}_z \right) - \beta \right] - \frac{\gamma \sigma^2}{2\sigma_W}$$

$$\hat{\sigma}^2_W = f^2\sigma^2_y\hat{n}_k^2 + \sigma^2_p\hat{n}_z^2$$

$$\frac{\hat{C}}{W} = \frac{1}{1 - \gamma}\left[ \beta - \gamma \left( r_k \hat{n}_k - (r_z + \varepsilon) \hat{n}_z \right) \right] + \eta \hat{n}_k + \frac{\gamma \sigma^2}{2\sigma_W}$$

where $r_k = (1 - \eta)f - \delta$.

---

12 Turnovsky and Chattopadhyay only sketch the solution procedure. For a more explicit explanation of the solution procedure for a problem of this type (though in a somewhat different context) see García-Peña and Turnovsky (2005).
$$r_z = 1 - \left[ \phi\left( h_1^T \right) e^{-rT} + \lambda (1 + \frac{1}{n_k}) \phi\left( h_2^T \right) \right] \frac{\int_0^T \left[ \phi\left( h_1 e^{-rs} + \lambda (1 + \frac{1}{n_k}) \phi\left( h_2 \right) \right) ds}{c_0}$$  

(13g)

$$h_1 = \left[ \frac{n_k \sqrt{f^2 \sigma_y^2 + \sigma_p^2}}{2} - \frac{r^*}{n_k \sqrt{f^2 \sigma_y^2 + \sigma_p^2}} \right] - \frac{\ln\left| n_z / \theta \right|}{\ln\left| n_z / \theta \right|}, h_2$$

$$= \left[ \frac{n_k \sqrt{f^2 \sigma_y^2 + \sigma_p^2}}{2} + \frac{r^*}{n_k \sqrt{f^2 \sigma_y^2 + \sigma_p^2}} \right] + \frac{\ln\left| n_z / \theta \right|}{\ln\left| n_z / \theta \right|}$$

and $h_i^T = h_i$ at time $t + T$ ($i = 1, 2$).

This equilibrium has the analogous structure to the equilibrium (12) of Turnovsky and Chatterpadhyay. The macroeconomic equilibrium is a stochastic growth path along which all real variables grow at the same stochastic rate. Equations (13a) and (13b), together with (13f) and (13g) jointly determine the equilibrium portfolio shares $h$, $n_z$ such that the risk-adjusted rate of return to capital equals the risk adjusted cost of debt. These expressions highlight the two determinants of the optimal portfolio shares. The first is the speculative component, which depends upon the expected differential between the return to capital and borrowing costs, while the second reflects the hedging behavior on the part of the investor and depends upon the relative variances associated with the returns on these two assets. Having obtained the optimal portfolio shares, (13d) determines the equilibrium variance along the balanced growth path, (13c) yields the equilibrium mean growth rate, and (13e) the consumption-wealth ratio, while (13g) determines the borrowing premium. Evaluating the integral (11) along the equilibrium path yields the welfare of the representative agent.$^{13}$

$$\Omega = \frac{\gamma}{\gamma - \eta f n_k}$$

(13h)

3.1. Calibration

When the economy encounters higher volatility, it affects the interest premium through changes in the demand for and supply of foreign debt. As a direct effect, an increase in volatility immediately shifts the debt supply curve to the left, as Fig. 3 illustrates. In response, the agent adjusts his portfolio shares, affecting the demand for foreign debt. The net effect is a combination of these two. We can observe similar effects on the volatility of wealth. In a debtor economy, the direct effect is destabilizing and the indirect effect from the portfolio adjustment is stabilizing. For example, consider an increase in external volatility, $\sigma_p^2$. From equation (13d), we can derive

$$\frac{\partial \sigma_W^2}{\partial \sigma_p} = 2n_z \sigma_p + 2 \left( f^2 \sigma_y^2 + \sigma_p^2 \right) \left( \hat{n}_k - \bar{n}_k \right) \left( \frac{\partial \sigma_W^2}{\partial \sigma_p} \right),$$

where $\hat{n}_k = \sigma_p / (f^2 \sigma_y^2 + \sigma_p^2)$ is the variance minimizing portfolio share of capital. The first term is the direct effect of a positive real exchange rate shock and the second term is an indirect effect of the portfolio adjustment. While the direct effect is always positive and destabilizing, the indirect effect depends on the sign of $\partial \sigma_W^2 / \partial \sigma_p$ since $\hat{n}_k - \bar{n}_k > 0$ for a debtor nation. We will see that, in our economy,

$^{13}$ In addition, the transversality condition $\lim_{t\to\infty} E(W^t e^{-rt}) = 0$ must hold. This can be shown to reduce to $C/W > \eta f n_k$. 

Please cite this article in press as: Chung, Keunsuk, Stephen J. Turnovsky, Foreign debt supply in an imperfect international capital market: Theory and evidence, Journal of International Money and Finance (2009), doi:10.1016/j.intmonfin.2009.06.015
the indirect effect is always negative and stabilizing. However, in most cases, we cannot determine analytically the signs of changes in the default risk premium, portfolio shares and the volatility of wealth. In this section, we conduct numerical analyses to consider how the agent’s resource allocation decision and the growth of wealth respond to various shocks to the economy. The list of parameters is in Table 1.

In Table 2(a) and (b), we report responses to volatility by comparing two separate economies: one with short-term debt (one year) and the other with long-term debt (10 years). The bold typefaced cases are our benchmarks. Table 2(c) summarizes similar responses to volatility in some selected variables, using the simplified version of the debt-supply curve employed by Turnovsky and Chatto-padhyay (2003). In our calibration, the benchmark debt-wealth ratios are 0.161 in the short-term debt economy and 0.194 in the long-term debt one, which are equivalent to 46.2% and 54.2% of debt-output ratios, respectively. This is generally consistent with the average external debt-output ratios over 1980 ~ 2005 period, which were 41.9% in low income countries, 55.8% in middle income countries and 33.6% in upper-middle income countries. For the two economies, the equilibrium interest rates range from 8.19% to 10.62% in our calibration. This too is generally consistent with the experience over the 1980 ~ 2005 period, where the average short-term and the long-term interest rates in developing countries were 11% and 12.36%, respectively. The mean growth rate of wealth is determined in 3 ~ 4% range in our model.

Rows in Table 2 track responses to the domestic output volatility shocks, and columns track responses to the external volatility. In the table, interest premium, mean growth rate and volatility of wealth are reported in percent. The welfare changes reported are expressed as percentage equivalent variations in the initial stock of wealth.

We start by considering effects of volatility on the supply and demand for foreign debt. An increase in volatility (either domestic or external) decreases the supply of foreign debt, generating an upward pressure on the interest premium. This, in turn, discourages the domestic agent’s foreign borrowing on the demand side. The overall effect on the interest premium depends on the relative size of the supply and demand shifts. In our calibration, a higher volatility of any kind decreases the interest premium in the new equilibrium. Increasing volatility discourages consumption as well, and hence deteriorates welfare. Although showing similar patterns, responses in equilibrium variables are different in magnitude, depending on the source of the shock. External volatility discourages the agent’s foreign borrowing and consumption more than domestic output volatility does, which implies a greater decline in the premium and a larger deterioration in welfare.

Depending on the size of the portfolio adjustment, the growth rate of wealth can be either stabilized or destabilized by a positive volatility shock. Recall that the effect of volatility on the volatility of wealth consists of a direct effect and an indirect portfolio adjustment effect. Therefore, more domestic output volatility, with a smaller portfolio adjustment, increases wealth volatility. However, external volatility causes a more substantial portfolio adjustment, and hence the effects are mixed. With respect to the mean growth rate of wealth, the overall effect of volatility is determined by two factors in our model. On the one hand, higher volatility negatively affects the growth rate by the reduction in foreign borrowing and investment. On the other hand, it may affect the growth rate positively if the volatility of wealth increases substantially. If the portfolio adjustment is small enough, the above two factors will help the economy grow faster. On the contrary, a sufficiently large portfolio adjustment may retard the growth of wealth. This is illustrated in Table 2(a) and (b). There we see that an increase in domestic volatility causes only a small degree of portfolio adjustment, and hence the growth rate increases; in contrast an increase in external volatility may retard the growth rate (in the long-term debt economy).

We can also compare responses of the equilibrium based on the length of the debt contract. When external volatility is high, the long-term debt economy borrows less than the short-term debt economy, but at a higher interest premium. By contrast, low external volatility allows the long-term

---

14 In the joint BIS–IMF–OECD–WB data, short-term external debt is defined as the external debt service within maturity of one year. Otherwise, it is considered as long-term external debt.

15 World Development Indicators, World Bank (2006). In our sample, the average external debt-GDP ratio is 37.3%.


Table 2

Effects of volatilities on equilibrium and the default-risk premium.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\sigma_p = 0.08$</th>
<th>$\sigma_p = 0.16$</th>
<th>$\sigma_p = 0.24$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_Y = 0.05$</td>
<td>$\eta_s$ $\psi_s$ $\omega_s$</td>
<td>$\eta_s$ $\psi_s$ $\omega_s$</td>
<td>$\eta_s$ $\psi_s$ $\omega_s$</td>
</tr>
<tr>
<td>$\sigma_Y = 0.10$</td>
<td>$0.196$ $2.373$ $2.384$</td>
<td>$0.162$ $3.345$ $3.123$</td>
<td>$0.131$ $3.398$ $3.577$</td>
</tr>
<tr>
<td>$\sigma_Y = 0.20$</td>
<td>$4.619$ $0.257$ $2.357$</td>
<td>$3.898$ $0.253$ $0.712$</td>
<td>$3.047$ $0.248$ $0.833$</td>
</tr>
<tr>
<td>$\sigma_Y = 0.30$</td>
<td>$0.194$ $3.359$ $3.904$</td>
<td>$0.161$ $3.423$ $4.327$</td>
<td>$0.130$ $3.470$ $4.606$</td>
</tr>
<tr>
<td>$\sigma_Y = 0.50$</td>
<td>$4.421$ $0.257$ $1.604$</td>
<td>$3.711$ $0.252$ $0$</td>
<td>$2.874$ $0.248$ $1.542$</td>
</tr>
<tr>
<td>$\sigma_Y = 0.80$</td>
<td>$3.635$ $0.253$ $1.369$</td>
<td>$2.969$ $0.249$ $-2.852$</td>
<td>$2.193$ $0.245$ $-4.390$</td>
</tr>
</tbody>
</table>

Note: 1. The premium function in the simplified model has the form:

$$\text{Prem} = \exp(\xi; \eta_s) - 1$$

where $\eta_s = 0.2269$, $\xi_{ST} = 0.2269$, $\xi_{LT} = 0.1771$.

2. Generally, responses of the variables to volatility shocks show similar patterns to those in panels (a) and (b).

Note: 1. $f = 0.3$, $\theta = 0.229$, $\lambda = 0.1$. Panels (a) and (b) compare the effects of volatility changes on equilibrium with a short-term ($T = 1$) and a long-term ($T = 10$) debt, respectively. a. An increase in any source of volatility discourages consumption and foreign borrowing (also investment). In all maturities, the external risk has a larger effect on the allocation of wealth than the domestic risk does. b. An increase in any source of volatility reduces the interest premium, due to the dominance of portfolio-adjustment over curve-shift. The range of portfolio adjustment is wider with a long-term debt, while the range of premium change is wider with a short-term debt, implying higher inelasticity in the supply of short-term debt. c. In the short-term debt economy, a positive volatility shock of any kind tends to accelerate growth of wealth and destabilize it. On the contrary, in the long-term debt economy, domestic volatility shock is growth-enhancing and destabilizing, while external volatility shock tends to stabilize the growth of wealth as the real exchange rate becomes highly volatile.

Note: 1. The premium function in the simplified model has the form: $\text{Prem} = \exp(\xi; \eta_s) - 1$ ($i = ST, LT$), $\eta_s = 0.2269$, $\xi_{LT} = 0.1771$. 2. Generally, responses of the variables to volatility shocks show similar patterns to those in panels (a) and (b). 3. However, due to the differences in shapes and volatility considerations, magnitudes of the responses in our current model and the simplified model are different. Compared with our current debt-supply model, (i) overall range of portfolio adjustment is wider in replicating the short-term debt economy; (ii) overall range of premium adjustment is narrower in the short-term, and wider in the long-term debt economy. a. In explaining domestic volatility effects on portfolio decision, the simplified model overstates the portfolio adjustments and understates the premium changes. b. In the ST debt economy, replicating the movement from medium to high external risk, the simplified model understates portfolio adjustments and overstates the premium changes. From medium to high external risk, it overstates the portfolio adjustments and understates the premium changes. c. In the LT debt economy, explaining the effect of increasing external risk, the simplified model understates the portfolio adjustments and overstates the premium changes.

debt economy to borrow more than its counterpart at a lower interest premium. It implies that the short-term debt supply curve is more elastic in the low range of debt size (hence a lower premium) while it becomes more inelastic as the debt size grows (higher premium). Irrespective of the origin of the volatility, adjustment with long-term foreign borrowing exceeds that with short-term borrowing. In the long-term debt economy, as the external volatility increases from low to medium, the volatility of wealth also increases. However, a further increase in external volatility stabilizes the growth of wealth. In contrast, in the short-term debt economy, the portfolio adjustment effect is always dominated by the direct effect, so any type of volatility shock is destabilizing.

Table 2(c) summarizes some selected equilibrium variables, of which the responses to shocks are based on the simplified model by Turnovsky and Chattopadhyay (2003). Foreign debt supply curve in the simplified model is a mildly convex exponential function of debt-wealth ratio of which the premium is determined by \( \omega(n_x) = e^{nx} - 1 \). The parameter value \( x \)'s in the short- and the long-term debt supply curves are set to replicate the benchmark equilibria in our representative economies.

Equilibrium responses in the simplified model show similar patterns as in our current model, although the magnitudes are different. When the simplified model overstates the portfolio adjustments compared with our current model, it simultaneously understates the premium changes, and vice versa. A key reason is the lack of volatility/maturity consideration in the simplified model. For instance, moving from the top-left to the bottom-right corner in Table 2 (i.e. from low-low to high-high risk), the simplified model overstates (understates) the portfolio adjustment, hence understating (overstating) the premium change in the short-term (long-term) debt economy. A closer look reveals some more contrasting details. In the simplified model, regardless of the maturity, the portfolio adjustment effect of a domestic volatility shock is overstated, and its effect on the interest premium is understated. In the short-term debt economy, replicating the movement from medium to low external risk, the simplified model understates portfolio adjustment and overstates the premium change. From medium to high external risk, it overstates the portfolio adjustment and understates the premium change. In the long-term debt economy, explaining the effect of increasing external risk, the simplified model understates the portfolio adjustment and overstates the premium change.

4. Empirical results

We now turn to some empirical evidence, where we examine the effects of volatility and relative size of foreign debt on the interest premium. Furthermore, we analyze whether or not the slope of foreign debt supply curve differs depending on the length of debt contract. Our regression equation is a linearized version of the interest premium function in equation (10).

As the first step, we construct the data for domestic and external volatilities by estimating volatilities of domestic output growth and real exchange rate depreciation. Conventionally, in cross-section regressions, these are measured by standard deviations of residuals from AR(1) processes of first-differenced log output and log real exchange rate. Since we obtain only one observation from each country by this approach, it poses a serious problem when the number of sample countries is not sufficiently large. Therefore we use an alternative approach for panel regressions. Each type of volatility is estimated by squared residuals of the following AR(1) processes:

\[
\Delta \log y_t = \phi_{y,0} + \phi_{y,1} \Delta \log y_{t-1} + u_{y,t}, \quad \Delta \log p_t = \phi_{p,0} + \phi_{p,1} \Delta \log p_{t-1} + u_{p,t},
\]

where \( u_{j,t} (j = y, p) \) denotes the residual from each of the above regressions at time \( t \). We can consider \( u_{j,t}^2 \) for each \( j \) as a conditional expectation of the true volatility parameter \( \sigma_j^2 \). Since these estimates of \( \sigma_j^2 \) involve measurement errors (denoted by \( \xi_{j,t} \)), we can write a set of equations

17 Although not reported, this stabilizing effect in the long-term debt economy becomes more dramatic as \( \eta \) increases.

18 Implied parameter \( x \)'s are 0.2269 and 0.1771 in the short-term and the long-term debt economies, respectively.

Please cite this article in press as: Chung, Keunsuk., Stephen J. Turnovsky,, Foreign debt supply in an imperfect international capital market: Theory and evidence, Journal of International Money and Finance (2009), doi:10.1016/j.jimonfin.2009.06.015
\[ \sigma^2_{y,t} = u^2_{y,t} + \gamma y_{t} \]
\[ \sigma^2_{p,t} = u^2_{p,t} + \gamma p_{t} \quad (15) \]

where \( E_{t-1}(\gamma_{j,t}) = 0 \) for each \( j (= y, p) \).

Define the dependent variable \( \text{Prem}_{t} = r_{2} - r_{1} \). It represents the interest premium derived in the model. Now write a linearized version of equation (10) as follows:

\[ \text{Prem}_{t} = \beta_{0} + \beta_{1} u^2_{y,t} + \beta_{2} u^2_{p,t} + \beta_{3} \left( \frac{PZ}{Y} \right)_{t} + v_{t} \quad (16) \]

where the debt-output ratio in equation (16) is used as a proxy for the debt-wealth ratio in equation (10).\(^{19}\) Substituting equation (15) into equation (16), we obtain

\[ \text{Prem}_{t} = \beta_{0} + \beta_{1} u^2_{y,t} + \beta_{2} u^2_{p,t} + \beta_{3} \left( \frac{PZ}{Y} \right)_{t} + v_{t} \quad (17) \]

where \( v_{t} = v_{t} + \gamma y_{t} + \gamma p_{t} \). Due to possible correlation between \( u^2_{j,t} \) variables and error terms, \( v_{t} \), we use one-period lags of the independent variables as instruments.

For the regressions, we use an unbalanced panel data of 20 developing countries in Africa, Asia, Europe and Latin America. The selection of countries in our sample is determined by the data availability. Table 3 reports the correlation coefficient matrix of the explanatory variables, in order to check the existence of multicollinearity. The coefficients are within reasonable ranges. Although the correlation between the short-term debt and the long-term debt is rather high, these two series do not enter the same regression equation. Therefore we use them as they are.\(^{20}\)

To control for the country-specific effects and time-fixed effects, we allow the intercepts to vary over individual countries, as well as over time. The test for no country-specific effects is rejected. For the time-fixed effects, we consider the Mexican crisis and Asian crisis, assuming the impacts of these two events to have been global.\(^{21}\) Thus we divide the sample period into three sub-periods: pre-Mexican crisis (1994q1–1994q3), post-Mexican/pre-Asian crisis (1994q4–1997q2) and post Asian crisis (1997q3–2004q2) periods.

This leads us to formulate the following system of equations:

\[ \text{Prem}_{it}^{ST} = \beta_{0t}^{ST} + \beta_{1t}^{ST} D_{2} + \beta_{2t}^{ST} D_{3} + \beta_{3t}^{ST} u^2_{y,it} + \beta_{4t}^{ST} u^2_{p,it} + \beta_{5t}^{ST} \left( \frac{PZ}{Y} \right)_{t} + v_{it}^{ST} \quad (18) \]
\[ \text{Prem}_{it}^{LT} = \beta_{0t}^{LT} + \beta_{1t}^{LT} D_{2} + \beta_{2t}^{LT} D_{3} + \beta_{3t}^{LT} u^2_{y,it} + \beta_{4t}^{LT} u^2_{p,it} + \beta_{5t}^{LT} \left( \frac{PZ}{Y} \right)_{t} + v_{it}^{LT} \]

where \( i = 1, 2, \ldots, N \) and \( t = 1, 2, \ldots, T \). As was noted, we allow the intercepts to vary over individual countries (indexed by country-subscript \( i \)) and over time (indexed by time-subscript \( t \)). \( \beta_{0t}^{ST} \) and \( \beta_{0t}^{LT} \) are the intercepts of country \( i \) in the short-term and the long-term equations, respectively. They depict country-specific characteristics that are not captured by the explanatory variables in our regressions, such as differences in government policies, commercial laws and other socioeconomic environments.\(^{22}\)

Analogously, \( \lambda_{t}^{ST} \) and \( \lambda_{t}^{LT} \) represent the time-fixed effects in the short-term and the long-term equations during each sub-period \( s \) that are common to all sample countries. \( D_{2} \) and \( D_{3} \) are the time dummies for the second and third sub-periods, respectively. We also allow the distribution of the error terms to vary over countries.\(^{23}\) \( v_{it}^{ST} \) and \( v_{it}^{LT} \) denote the error terms in the short-term and the long-term equations

\(^{19}\) Note that, on the equilibrium balanced growth path, all real variables (including capital stock, output, foreign debt and wealth) grow at the same rate. Also note that for the linear technology our model employs, \( (PZ/W) = f(PZ/Y) \) holds.

\(^{20}\) We also conduct augmented Dickey–Fuller unit root tests. The null hypotheses of unit roots are rejected in all series except the long-term debt. Since our model uses the stock of debt as a measure of external debt, we use it without entering them in the first difference.

\(^{21}\) We consider only variable intercepts over different sub-periods, but not variable coefficients.

\(^{22}\) To examine sovereign risks of 13 EU countries, Lemmen and Goodhart (1999) use variables such as government’s tax raising capability, expenditure, inflation rate and its variability. We focus only on the variables in our theoretical model.

\(^{23}\) It is natural to assume variability of intercepts and error terms over individual countries because economically or geographically distant two countries are not very likely to share the same characteristics.

Please cite this article in press as: Chung, Keunsuk., Stephen J. Turnovsky,, Foreign debt supply in an imperfect international capital market: Theory and evidence, Journal of International Money and Finance (2009), doi:10.1016/j.jimonfin.2009.06.015
for country $i$ at time $t$. $\beta_1$ and $\beta_2$ capture the effects of changes in the output volatility and the RER volatility, respectively ($j = ST, LT$). The coefficient of the debt-GDP ratio, $\beta_3$ is the slope of debt supply curve. In equation (10), the domestic borrower’s equilibrium portfolio choice is one of the determinants of the interest premium. Therefore, we investigate the interactions between risk and the size of debt, by including product terms of volatility parameters and the relative debt size. One of the predictions of our analytical model is that, depending on the risk parameters and the relative size of debt, the short-term debt supply curve is more inelastic (hence steeper) than the long-term curve over a reasonable range of debt size. We will pay special attention to this aspect throughout this section. 

4.1. Data

We use unbalanced quarterly panel data from 20 small open developing economies over 1994q1 through 2004q2 period. Included economies are: Brazil, Bulgaria, Chile, Cyprus, Czech Republic, Hong Kong, Hungary, Indonesia, Korea, Lithuania, Malaysia, Malta, Mexico, Philippines, Poland, Singapore, Slovenia, South Africa, Taiwan, and Thailand. The data description and their sources are as follows:

1. **External debt**: Short-term and long-term external debt data are collected from joint BIS–IMF–OECD–World Bank Data on External Debt. Short-term debts are defined as the sum of government securities with less-than-a-year maturities, while long-term debts are the ones of which the maturities are longer than a year.

2. **Real GDP**: Real GDP series are denominated in terms of national currencies. Most of the real GDP data are collected from International Financial Statistics (IFS) Data CD. Data for Chile, Cyprus, Mexico, South Africa, and Taiwan are collected from individual countries’ central bank websites. Singapore data are collected from Singstat. Due to the lack of quarterly GDP deflator data series, nominal GDP of each country is denominated by quarterly CPI.

3. **Real exchange rate**: We construct the real exchange rate by calculating nominal exchange rate (national currency to US dollar) multiplied by the ratio of the US price index to the domestic price index. All nominal exchange rates and price indices are collected from IFS Data CD, except Taiwan. Taiwan data are collected from the Department of Statistics of Taiwan.

4. **Volatilities**: Our primary measure of output growth volatility is being defined as conditional expectation of the variance of output growth, the squared residual from the AR(1) regression of log difference in output. The volatility of real exchange rate depreciation is defined analogously.

5. **Interest rates**: Interest rate series are collected from various sources, including IFS Data CD, data releases from individual countries’ central banks, ministries of finance, and departments of statistics. All series are market interest rates on government bills (with maturity shorter than one year—mostly three month T-bills) and bonds (with maturity longer than one year—mostly fifteen year G-bonds), except for the long-term interest rate of Brazil. Brazil’s long-term interest rate series used in the regression are interest rates on the long-term loans by Brazilian central bank to domestic firms (TJLP). We use 6-month LIBOR for the risk-free world interest rate. The default-risk premia are the differences between individual countries’ short-/long-term interest rates and the 6-month LIBOR.

---

Table 3

<table>
<thead>
<tr>
<th></th>
<th>RER VOL</th>
<th>(Debt/GDP)_{ST}</th>
<th>(Debt/GDP)_{LT}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output VOL</td>
<td>0.0027</td>
<td>-0.0107</td>
<td>-0.0419</td>
</tr>
<tr>
<td>RER VOL</td>
<td>-0.0107</td>
<td>0.0419</td>
<td></td>
</tr>
<tr>
<td>(Debt/GDP)_{ST}</td>
<td>0.0419</td>
<td>-0.0441</td>
<td></td>
</tr>
<tr>
<td>(Debt/GDP)_{LT}</td>
<td>-0.0441</td>
<td>0.5733</td>
<td></td>
</tr>
</tbody>
</table>

---

Please cite this article in press as: Chung, Keunsuk., Stephen J. Turnovsky,, Foreign debt supply in an imperfect international capital market: Theory and evidence, Journal of International Money and Finance (2009), doi:10.1016/j.jimonfin.2009.06.015
4.2. Results

Table 4 summarizes the main results of our regressions. Because of space limitations we report only the regression coefficients relating to the structural characteristics of the model, though we briefly summarize some of the fixed effects, which are available in an expanded version of the paper. The first column reports results for the basic regression specified by equation (18). Both for the short-term borrowing premium and the long-term borrowing premium the debt to GDP ratio is highly significant. Moreover, the coefficient in the short-term equation is approximately twice that in the long-term equation, suggesting that short-term debt supply is more inelastic than is long-term debt, consistent with our numerical simulations and the patterns illustrated in Fig. 2. In addition, both domestic and external volatilities have their predicted signs, raising the borrowing premiums on both short-term and long-term debt. Both sources of volatility are highly significant in the case of long-term debt; in the case of short-term debt external volatility remains highly significant and internal volatility less so. In both cases, external risk has a larger impact on the premium than does the domestic risk, consistent with Fig. 3-1 and 3-2, and the underlying numerical simulations.

Our model shows that any changes in volatility or the relative debt size will affect the balanced growth path equilibrium. Such an effect in turn will influence the position of the debt supply curve in equation (10). To incorporate this aspect, we introduce the interactions between volatility measures and the relative size of debt in column (2). The last column uses the same regression equation as in column (2), but we omit the statistically insignificant independent variables.

Coefficients of the interaction terms show mixed responses in their signs and significance. The coefficients of $\sigma^2_s$ and $\sigma^2_s \times (\text{Debt}/\text{GDP})$ terms are insignificant in the short-term equations, while the coefficients of $\sigma^2_s$ and $\sigma^2_s \times (\text{Debt}/\text{GDP})$ remain significant both in the short-term and long-term equations. The coefficients for any types of volatility alone are all positive, while the interaction terms for any types of volatility mostly have negative coefficients. In the short-term equation, for a higher external volatility, the effect of the debt size on the interest premium is significantly larger. Similarly, for a larger debt-wealth ratio, increasing external volatility causes a further shift in the debt supply curve. It is consistent with the illustrations in Fig. 3-2. In contrast, coefficients of all the other interaction terms suggest the opposite implications, which do not fit the predictions of the model. One explanation can be that increasing volatility may have a predominantly large negative impact on the domestic agent’s borrowing decision, reducing the balanced growth path.

Table 4

<table>
<thead>
<tr>
<th>Dependent variable = Prem</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Short-term</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output VOL</td>
<td>0.1775 (0.1058)*</td>
<td>0.1911 (0.1640)</td>
<td></td>
</tr>
<tr>
<td>RER VOL</td>
<td>1.0435 (0.0842)**</td>
<td>0.9193 (0.1751)**</td>
<td>0.6727 (0.1743)**</td>
</tr>
<tr>
<td>Debt/GDP</td>
<td>0.1224 (0.0136)**</td>
<td>0.1117 (0.0171)**</td>
<td>0.0971 (0.0147)**</td>
</tr>
<tr>
<td>Output VOL x Debt/GDP</td>
<td>-0.4337 (1.7743)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RER VOL x Debt/GDP</td>
<td>5.1716 (3.0410)*</td>
<td>10.4404 (2.8554)**</td>
<td></td>
</tr>
<tr>
<td><strong>Long-term</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output VOL</td>
<td>0.2413 (0.1013)**</td>
<td>0.3699 (0.1549)**</td>
<td>0.4044 (0.1566)**</td>
</tr>
<tr>
<td>RER VOL</td>
<td>0.7502 (0.0594)**</td>
<td>0.8331 (0.1270)**</td>
<td>0.7816 (0.1312)**</td>
</tr>
<tr>
<td>Debt/GDP</td>
<td>0.0603 (0.0029)**</td>
<td>0.0662 (0.0033)**</td>
<td>0.0667 (0.0033)**</td>
</tr>
<tr>
<td>Output VOL x Debt/GDP</td>
<td>-0.7713 (0.2933)**</td>
<td>-0.7148 (0.3192)**</td>
<td></td>
</tr>
<tr>
<td>RER VOL x Debt/GDP</td>
<td>-0.6557 (0.3039)**</td>
<td>-0.5766 (0.3171)**</td>
<td></td>
</tr>
</tbody>
</table>

Note: (1) $\beta'_{ij} = \beta_{p_{ij}} + \sum_{j=1}^{3} \beta_{p_{ij}} D_{it} + \beta_{p_{ij}} \sigma^2_{p_{ij}} + \beta_{p_{ij}} \sigma^2_{p_{ij}} + \beta_{p_{ij}} (\text{PZ} / \text{Y}_{it} + \nu_{it})$

(2) $\beta'_{ij} = \beta_{p_{ij}} + \sum_{j=1}^{3} \beta_{p_{ij}} D_{it} + \beta_{p_{ij}} \sigma^2_{p_{ij}} + \beta_{p_{ij}} \sigma^2_{p_{ij}} + \beta_{p_{ij}} (\text{PZ} / \text{Y}_{it} + \nu_{it})$

where $j = \text{ST}$ and $\text{LT}$.

(3) The same as Model (2) specification. Insignificant variables are omitted.

1. * and ** indicate that the coefficients are significantly different from zero at 10%, 5% and 1% level, respectively.
2. The numbers in (parentheses) are standard errors.
debt–wealth ratio. Our calibration in Table 2 also shows that, increasing volatility (either domestic or external) reduces the equilibrium share of foreign debt and the interest premium. Wald coefficient tests conducted for the slope difference reject the null hypothesis of equal slope, implying that the short-term debt supply curve is more inelastic (steeper slope) than is the long-term debt supply curve, consistent with Fig. 2.25

Regarding time-fixed effects, in all models the Mexican crisis affects the short-term and the long-term interest premia positively, while the Asian crisis has negative effects on them in both maturities. These effects are all significant. Considering its impact on the global economy, the coefficient signs of the Asian crisis dummies are counter-intuitive. However, this could be a result of debt payment rescheduling and sharp decreases in volatilities after the Asian crisis in the inflicted economies. For most of the countries, the intercepts are significantly different from zero, which means that the interest premia are positive in those countries. Also the Wald coefficient test shows that individual countries have different intercepts.26

5. Conclusions

The fact that developing economies are subject to default-risk and hence have limited access to the international credit market is an issue of great concern to both researchers and policymakers. Macrodynamic models have tended to incorporate this fact by assuming that such economies face an upward sloping supply of debt, whereby their borrowing costs increase with their debt position. While this formulation has appeal as a plausible reduced form equilibrium relationship, it is essentially ad hoc. In this paper we have developed a model of foreign debt supply, which incorporates the rational behavior of participants in the foreign debt market, in light of the risks they face. Using this framework we derive the foreign-debt supply curve as an upward sloping function of the relative size of debt, measured by debt-wealth ratio. In addition, the foreign-debt supply curve is characterized by the volatility of domestic output growth (internal volatility) and the volatility of bond prices (external volatility), together with their interaction with the debt-wealth ratio. Second, the borrowing premium depends upon the length of the debt contract, with the curvature of the foreign debt supply curve varying inversely with its length. Third, the external source of volatility exerts a larger effect on the agent’s portfolio/consumption decision and his welfare than does internal volatility. The growth rate of wealth and its volatility can either increase or decrease, depending on the direct effect of a shock and the indirect portfolio adjustment effect.

Using an unbalanced panel of 20 countries over 40 quarters, we find evidence that both the short-term and the long-term interest premia are positively related with the relative size of debt, measured by debt-GDP ratio. Furthermore, we find that the short-term debt supply curve is more inelastic than the long-term curve in the reasonable range of debt size. Also, we examine the effects of a change in domestic and external risk on the short-term and the long-term interest premia. We find that the real exchange rate volatility has a strong, positive effect on the interest premia in all specifications in our regressions, while the effect of the domestic output volatility is weaker. We also find that an increasing volatility may affect the slope of the debt supply curve as well as its position by interacting with the relative debt size. Overall, we find that this empirical evidence offers encouraging support for the underlying theoretical model.

Acknowledgements

This paper is a revised version of Chapter 1 of Keunsuk Chung’s thesis written at the University of Washington. Chung’s research was supported in part by the Grover and Creta Ensley Fellowship and

25 The logarithmic version of the regression referred to footnote 24 yields a similar pattern, implying that the short-term curve is more convex than the long-term curve.

26 In an expanded version of this paper we have also run regressions allowing coefficients to vary over country groups based on the level of indebtedness. Major properties of the foreign debt supply curve obtained in the entire sample regressions are well preserved in the sub-group regressions as well. In addition, slope comparison among different indebtedness groups suggests the existence of non-linearity in the foreign debt supply curve.

Please cite this article in press as: Chung, Keunsuk., Stephen J. Turnovsky,. Foreign debt supply in an imperfect international capital market: Theory and evidence, Journal of International Money and Finance (2009), doi:10.1016/j.jimonfin.2009.06.015
Appendix. Derivation of expected future profit of the international financial intermediaries

When the interest rate, \( r_x \), on risky debt is different from \( r^* \), the present value of its expected future payments, discounted by \( \rho = r^* + \epsilon \), will not equal the value of initial borrowing, \( \langle PZ \rangle_t \), if we use the natural probability measure. Therefore, evaluating the risky debt using the expected present value under the natural probability will leave arbitrage opportunities via trading it with some other assets under the natural probability measure. Therefore, evaluating the risky debt using the expected present value under the natural probability will leave arbitrage opportunities via trading it with some other assets (e.g. risk-free bonds with a deterministic interest rate \( \rho \)). Suppose that there is a probability measure that is equivalent to the natural one, and under which the present value of expected total future payments from the risky debt is a martingale. Under such a probability measure, the best estimate at time \( t \) for the present value of payoff from the risky debt will be the current value of the initial borrowing. In the asset-pricing literature, it is known that the existence of such a probability measure prevents arbitrage between the risky asset and the risk-free bond. The same argument is true for the intermediary's payoff to the foreign depositors. If the expected present values of the payoffs – from domestic borrowers and to foreign depositors – under the risk-neutral probability are martingales, and if the intermediary's lending (the revenue) is financed by no other sources than the deposits (the cost), the present value of its expected profit will be zero.

Rewrite the basis of debt repayment at time \( t + s \) as follows:

\[
\left( \frac{\langle P \rangle_t}{Z_t} \right)_{t+s} = \langle P \rangle_{t+s}, \quad I_{t+s} + \lambda K_{t+s} \left( 1 - I_{t+s} \right)
\]

The condition for full repayment, \( A_{t+s} \), is rewritten as

\[
\theta W_t e^{\frac{r^* - \rho}{2} \sigma_t^2} \left[ w(t+s) - \hat{w}(t) \right] \geq \langle P \rangle_t e^{(r^* - \frac{1}{2} \sigma_t^2)p(t+s) - p(t)} \quad \text{or}
\]

\[
\theta W_t e^{\frac{r^* - \rho}{2} \sigma_t^2} \left[ p(t+s) - p(t) \right] \geq \langle P \rangle_t e^{(r^* - \frac{1}{2} \sigma_t^2)p(t+s) - p(t)}
\]

Where \( \theta W_t \) is the change in the value of the intermediary's payoff to foreign depositors, \( Z_t \) is the present value of the expected total future payments from the intermediary's lending, \( y(t) \) is the expected future payoffs from the intermediary's lending, \( \hat{w}(t) \) is the expected future value of the intermediary's payoff to foreign depositors, \( \langle P \rangle_t \) is the present value of the expected total future payments from the intermediary's lending, \( r^* \) is the risk-free rate, \( \rho \) is the cost of capital, \( \sigma_t^2 \) is the variance of the return on the intermediary's lending, \( p(t) \) is the present value of the expected total future payments from the intermediary's lending, and \( A_{t+s} \) is the condition for full repayment.

Assume that the financial intermediary takes the equilibrium growth of domestic wealth as given. Let

\[
B_y(t) = \frac{y(t)}{\sigma_y}, \quad B_p(t) = \frac{p(t)}{\sigma_p}
\]

where \( \rho = r^* + \epsilon \). Then condition (A.1) reduces to

\[
\theta W_t e^{(r^* - \frac{1}{2} \sigma_t^2)\sigma_{t+s} \sigma_{t+s} \sigma_{t+s} \sigma_{t+s}} \left[ w(t+s) - \hat{w}(t) \right] \geq \langle P \rangle_t e^{(r^* - \frac{1}{2} \sigma_t^2)p(t+s) - p(t)}
\]

Take logarithms, and let \( z_1 = \frac{1}{B_y(t+s) - B_y(t)} \sqrt{\sigma_2} \) and \( z_2 = \frac{1}{B_p(t+s) - B_p(t)} \sqrt{\sigma_2} \) where \( z_1, z_2 \sim N(0, 1) \) and uncorrelated with each other. We obtain the condition for \( A_{t+s} \) as follows:

\[
f \sigma_y \sqrt{z_2} + \sigma_p \sqrt{z_1} + \left( \frac{1}{2n_k} (\sigma_w^2 - \sigma_p^2) - \frac{\rho}{\sigma_p} \right) s + \frac{1}{n_k} \theta W_t \left( \frac{\langle P \rangle_t}{C_1} \right)
\]

In an analogous manner, the condition for \( A_{t+s}^C \) can be obtained.

\[\int^{t+s} (y - \rho) / n_k \sigma_y ds \leq \infty \text{ and by Girsanov theorem, there exists a risk-neutral probability equivalent to the natural one under which } B_y(t+s) - B_y(t) \text{ and } B_p(t+s) - B_p(t), \text{ defined above, are standard Brownian motions.} \]

Please cite this article in press as: Chung, Keunsuk., Stephen J. Turnovsky,, Foreign debt supply in an imperfect international capital market: Theory and evidence, Journal of International Money and Finance (2009), doi:10.1016/j.jimonfin.2009.06.015
With the above conditions, we can simplify the expression for the expected present value of the debt payment subject to the risk of default. From equation (6), the expression for \( E_t[\{ PZ \}_{t+s}] \) can be rewritten

\[
E_t[\{ PZ \}_{t+s}] = E_t[(PZ)_{t+s} I_{A_{t+s}} + \lambda K_{t+s} (1 - I_{A_{t+s}})]
\]

\[
= E_t[(PZ)_t e^{-\frac{z^2}{2}\sigma_t^2} s + \sigma_t [B_p(t,s) - B_p(t)] I_{A_{t+s}} + \lambda K_t e^{-\frac{z^2}{2}\sigma_t^2} s + \sigma_t [B_p(t,s) - B_p(t)] - \bar{\sigma}_z [B_p(t,s) - B_p(t)] (1 - I_{A_{t+s}})]
\]

\[
= \int_{A_{t+s}} (PZ)_t e^{-\frac{z^2}{2}\sigma_t^2} s + \sigma_t [B_p(t,s) - B_p(t)] I_{A_{t+s}} + \lambda K_t e^{-\frac{z^2}{2}\sigma_t^2} s + \sigma_t [B_p(t,s) - B_p(t)] - \bar{\sigma}_z [B_p(t,s) - B_p(t)] (1 - I_{A_{t+s}})
\]

\[
\times \frac{1}{2\pi} e^{-\frac{z_1^2 + z_2^2}{2}} dz_1 dz_2 + \int_{A'_{t+s}} \lambda K_t e^{-\frac{z^2}{2}\sigma_t^2} s + \sigma_t [B_p(t,s) - B_p(t)] (1 - I_{A_{t+s}})
\]

\[
\times \frac{1}{2\pi} e^{-\frac{z_1^2 + z_2^2}{2}} dz_1 dz_2
\]

The first term becomes

\[
E_t[\{ PZ \}_{t+s}] = (PZ)_t e^{is} \int_{A_{t+s}} \frac{1}{2\pi} e^{-\frac{(z_1 - \sigma_p \sqrt{s} z_3)^2}{2}} \frac{1}{2\pi} e^{-\frac{z_2^2}{2}} dz_1 dz_2 + \lambda K_t e^{is} \int_{A'_{t+s}} \frac{1}{2\pi} e^{-\frac{(z_1 - \sigma_p \sqrt{s} z_3)^2}{2}} \frac{1}{2\pi} e^{-\frac{z_2^2}{2}} dz_1 dz_2
\]

(A.2)

Let \( z'_1 = z_1 - \sigma_p \sqrt{s} z_3 = z_1 + \bar{n}_z \sigma_p \sqrt{s} \) and \( z_4 = z_2 - \bar{n}_k f \sigma_y \sqrt{s} \). Then

\[
E_t[\{ PZ \}_{t+s}] = (PZ)_t e^{is} \int_{A_{t+s}} \frac{1}{2\pi} e^{-\frac{z'_1^2}{2}} \frac{1}{2\pi} e^{-\frac{z_2^2}{2}} dz'_1 dz_2 + \lambda K_t e^{is} \int_{A'_{t+s}} \frac{1}{2\pi} e^{-\frac{z'_1^2}{2}} \frac{1}{2\pi} e^{-\frac{z_2^2}{2}} dz'_1 dz_2
\]

(A.3)

The conditions for \( A_{t+s} \) and \( A'_{t+s} \) are, respectively,

\[
f\sigma_y \sqrt{s} z_2 \leq \sigma_p \sqrt{s} z'_1 + \frac{\bar{n}_k}{2} \left( f^2 \sigma_y^2 + \sigma_p^2 \right) s - \frac{r^* s}{n_k} + \frac{1}{n_k} \ln \left( \frac{PZ}{t} \right) \]

(A.4)

\[
f\sigma_y \sqrt{s} z_4 \leq \sigma_p \sqrt{s} z_3 - \frac{\bar{n}_k}{2} \left( f^2 \sigma_y^2 + \sigma_p^2 \right) s - \frac{r^* s}{n_k} + \frac{1}{n_k} \ln \left( \frac{PZ}{t} \right) \]

(A.5)
To simplify the expression, we apply the following transformations:

\[
\begin{pmatrix}
    x_1 \\
    x_2
\end{pmatrix} = \begin{pmatrix}
    \frac{f \sigma_s \sqrt{s}}{\sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}} & \frac{\sigma_p \sqrt{s}}{\sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}} \\
    \frac{f \sigma_s \sqrt{s}}{\sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}} & \frac{f \sigma_s \sqrt{s}}{\sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}}
\end{pmatrix} \begin{pmatrix}
    z'_1 \\
    z'_2
\end{pmatrix} = \begin{pmatrix}
    x_3 \\
    x_4
\end{pmatrix}\]

(A.4) and (A.5) thus become

\[
x_2 \geq \frac{\tilde{n}_k}{2} \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s} - \frac{r^* s}{\tilde{n}_k \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}} + \frac{\ln \left( \frac{PZ_{0}}{\eta_0} \right)}{\tilde{n}_k \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}} \tag{A.6}
\]

\[
x_4 < \frac{\tilde{n}_k}{2} \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s} - \frac{r^* s}{\tilde{n}_k \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}} + \frac{\ln \left( \frac{PZ_{0}}{\eta_0} \right)}{\tilde{n}_k \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}} \tag{A.7}
\]

Let \( h_1 = -\frac{\tilde{n}_k \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}}{2} - \frac{r^* s}{\tilde{n}_k \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}} - \frac{\ln \left( \frac{PZ_{0}}{\eta_0} \right)}{\tilde{n}_k \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}} \) and

\[
h_2 = -\frac{\tilde{n}_k \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}}{2} + \frac{r^* s}{\tilde{n}_k \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}} + \frac{\ln \left( \frac{PZ_{0}}{\eta_0} \right)}{\tilde{n}_k \sqrt{(f^2 \sigma_y^2 + \sigma_p^2) s}}
\]

Substituting the conditions (A.6) and (A.7) into equation (A.2), we obtain

\[
E_t \left[ (PZ)_{t+s} \right] = (PZ)_t e^{s \Phi(h_1)} + \lambda K_t e^{\eta T} \Phi(h_2) \tag{A.8}
\]

\[
E_t \left[ (PZ)_{t+T} \right] = (PZ)_t e^{T \Phi(h_1^T)} + \lambda K_t e^{T \Phi(h_2^T)} \tag{A.9}
\]

Substituting (A.8) and (A.9) into equation (6) yields equation (7).

References


Please cite this article in press as: Chung, Keunsuk., Stephen J. Turnovsky,, Foreign debt supply in an imperfect international capital market: Theory and evidence, Journal of International Money and Finance (2009), doi:10.1016/j.jimonfin.2009.06.015


