Foreign capital inflows: direct investment, equity investment, and foreign debt

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Abstract: We develop a two-country stochastic growth model with production, relative price and sovereign default risks. Domestic production and relative price volatilities cause more fluctuations in the agents’ portfolio decisions than the volatility of Foreign Direct Investment (FDI) production does. Both the sovereign risk and separability of FDI capital affect the composition of foreign capital inflows in two directions. The direct effect induces substitution of FDI for more Foreign Portfolio Investment (FPI) and foreign borrowing, while the indirect effect encourages FDI due to the increase in FDI’s marginal contribution to the foreign agent’s welfare after default.

Keywords: FDI; foreign direct investment; foreign equity investment; inelastic debt supply; volatility.


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1 Introduction

Capital flows to developing countries, although they may be small relative to the volume of entire cross-border capital flows, have been growing rapidly both in volume and in variety. The biggest proportion of such flows is still in the form of foreign debt, while other types of capital flows are growing in shares. Considerable literature analyses the causes of foreign capital flows and their effects on growth and welfare of the host country. However, such literature has focused on each type of foreign capital in isolation, or considered that distinction among types of foreign capital flows does not yield much policy implications in the beliefs that each type can easily replace one another and that foreign investors can get around the control problems with financial derivatives and
other instruments. However, recent empirical evidence shows that different types of capital (particularly Foreign Direct Investment, in short, FDI) have different influences on investment and consumption (Borensztein et al., 1998; Bosworth and Collins, 1999; Reisen and Soto, 2001). Some of the theoretical models explicitly incorporating different types of foreign capital flows are Albuquerque (2003), Kraay et al. (2004) and Goldstein et al. (2008). Albuquerque (2003), focusing on imperfect enforcement of financial contracts and FDI capital’s inalienability, shows that FDI capital flows are less volatile than other capital flows. Kraay et al. (2004) develops a model of North-South capital flows with diminishing returns, production risk and default risk, and argues that inefficient renegotiation during a crisis period can explain the dominance of loans among international capital flows. From the source country’s perspective, Goldstein et al. (2008) provides a theoretical model and empirical evidence showing that, with a higher probability of aggregate liquidity crises and less transparency in the source country’s capital market, the source county investors export more Foreign Portfolio Investment (FPI) than FDI. These models incorporate various measures of risk: an aggregate shock, the volatility of output growth as a measure of production risk, probability of sovereign default or the probability of liquidity crises. However, none of them take the volatility of real exchange rate or relative price into consideration, while it is one of the most crucial sources of risk in international portfolio decisions.

In this paper, we develop a model of stochastically growing open economies where the agents make portfolio decisions on investment and foreign debt in the presence of production, relative price and sovereign default risks. We classify the foreign capital flows in three categories: foreign debt, FDI and FPI. International lenders and international investors differ in that the lenders receive interest payments on foreign debt, while the investors receive returns to capital based on their shares. In contrast, FDI and FPI investors are distinguished based on the ownership and operational control over the capital. FPI investors, based on their equity shares, have partial ownership without operational control, whereas FDI investors hold both the ownership and operational control. Therefore, the share of each type of foreign capital flows is determined by its return-risk characteristics. As sources of the risk, we incorporate volatilities of domestic output, FDI output and the relative price of international bonds, as well as the probability of default, upon which the host country expropriates all types of foreign capital. The purpose of this paper is to see how structural changes, such as volatility shocks, changes in the probability of default and separability of FDI capital affect the domestic agent’s portfolio decision, composition of foreign capital inflows, growth of wealth and its volatility in the host economy.

This paper is organised as follows. In Section 2, we setup a stochastic dynamic general equilibrium model of two countries in which the home is a small open developing economy and the foreign is a large open one. In Section 3, we derive balanced-growth path equilibria in both countries and discuss some properties of the equilibrium portfolio shares. Section 4 reports results from calibration exercises, and Section 5 concludes.

2 The model

The world economy comprises two countries, the home and the foreign. Both domestic and foreign agents own and operate firms in the home country. The domestic agents hold a partial ownership in the domestic firms, whereas the foreign agents hold a partial
ownership in the domestic firms and full ownership in the foreign firms. The former type of foreign investment represents FPI, whereas the latter type represents FDI. Each firm hires domestic labour and combines it with capital to produce a single final product, which can be either consumed or invested at no cost of adjustment. The production functions of both firms are Cobb-Douglas and have the following forms:

\[
dY_d = (L_d K_d^{1-\alpha}) (dt + dy_d)
\]

\[
dY_f = A(L_f K_f^{1-\alpha}) (dt + dy_f).
\]

Subscripts \(d\) and \(f\), respectively, represent domestic- and foreign-owned variables whereas subscript \(A\) represents an aggregate variable, which is the sum of both the domestic and foreign ones. Once the allocation of domestic labour between the domestic and foreign firms, \(L_i (i = d, f)\), is determined out of inelastic aggregate supply, \(L_A\), it is combined with the aggregate capital, \(K_A\), and enters the production process as efficiency units. \(K_i\) represents the physical capital employed by country \(i\)’s firm. Each firm’s production process consists of a deterministic component and a stochastic one. The stochastic component, \(dy_i\), is a Brownian motion process, distributed \(N(0, \sigma_i^2 \, dt)\). The term \(A\) in the foreign production function can represent technological advantage or tax incentives by the home country government on FDI.

Before the stochastic change in production is realised, wage, \(a_dt\), is set by the marginal product of labour on the deterministic component of production.

\[
a_d \, dt = \alpha L_d^{\alpha} \left( \frac{K_d}{L_d} \right)^\alpha \left( \frac{K_d}{L_d} \right)^{1-\alpha} \, dt
\]

\[
a_f \, dt = A \alpha L_f^{\alpha} \left( \frac{K_f}{L_f} \right)^\alpha \left( \frac{K_f}{L_f} \right)^{1-\alpha} \, dt.
\]

If the domestic labour market is perfectly competitive, wages in both firms are equalised, thereby determining capital-labour ratio in each firm by

\[
\frac{K_d}{L_d} = \frac{A' L_d + L_f}{A' L_d} \left( \frac{K_d}{L_d} \right)^{1-\alpha} \left( \frac{K_f}{L_f} \right)^{1-\alpha}
\]

where \(A' = A^{1(1-\alpha)}\). We can rewrite equation (2) as follows:

\[
a_d \, dt = \alpha L_d^{\alpha} \left( \frac{A' L_d + L_f}{A'} \right)^\alpha \left( \frac{K_d}{L_d} \right) \, dt = \alpha f \, dt
\]

\[
a_f \, dt = A \alpha L_f^{\alpha} A' (A' L_d + L_f)^{1-\alpha} \left( \frac{K_f}{L_f} \right) \, dt = \alpha f^* \, dt.
\]

If \(A = 1\), \(f = f^* = L_d^\alpha\).

The returns to domestic and FDI capital are determined as residuals from production after paying for labour and depreciation. Therefore, any unanticipated changes in production will be reflected in the changes in capital returns.
Foreign capital inflows

\[ dR_d = \frac{dY_d - a_dL_d dt - \delta dt}{K_d} = r_d dt + f dy_d \]  
\[ dR_f = \frac{dY_f - a_fL_f dt - \delta' dt}{K_f} = r_f dt + f' dy_f \]

where \( r_d = \{(1 - \alpha)f - \delta\}, \) \( r_f = \{(1 - \alpha)f' - \delta'\}, \) and \( \delta(\delta') \) denotes the depreciation rate in the domestic (foreign) firm, respectively.

A part of domestic investment is financed by foreign debt, \( Z. \) For such borrowing, the domestic agent issues international bonds in the world credit market. We assume perpetuities for these bonds and each unit sells at a price, \( P, \) in terms of the final good. Process of this relative price is a geometric Brownian motion

\[ \frac{dP}{P} = \bar{\epsilon} dt + dp \]

where \( \bar{\epsilon} \) is the mean rate of depreciation in the international bond price, and \( dp \) is a Brownian motion process distributed \( N(0, \sigma^2 dt). \) Therefore, for a given amount of foreign debt, \( Z, \) the evolution of its value, \( PZ, \) is not only influenced by the interest rate charged to the debt, but also by the deterministic and stochastic changes of the bond price, \( P. \) We define the interest rate on foreign bonds as

\[ dR^*_t = r^*_t dt + dp = (r^* + \bar{\epsilon} + \lambda)dt + dp. \]

The mean interest rate is the sum of the world risk-free interest rate, \( r^*, \) the average depreciation rate of the bond price, and a risk premium, \( \lambda. \) \( \lambda \) denotes the arrival rate of a default or expropriation of foreign capital in a given interval of time \( dt. \) With this risk, a foreign creditor charges a constant premium of \( \lambda. \) Since the foreign debt is a perpetuity, the domestic agent continuously pays the foreign creditor a deterministic mean interest rate, \( r^*, \) and a stochastic rate, \( dp, \) over a time interval \( dt. \)

The domestic agent owns and operates the domestic firm, of which the capital, \( K_d, \) is financed partially by domestic wealth, partially by foreign borrowing, and partially by equity investment (FPI) from the foreign agent. An FPI investor has claims on the domestic production based on the shares he holds, but does not have the right to operate the domestic firm, whereas a FDI investor has the full ownership and control over the FDI firm.1 A unit of equity in the domestic firm, \( e_p, \) which sells at a price \( v_p, \) gives the foreign investors the net flow of production generated by one unit of domestic capital.2 Then the domestic agent’s wealth is determined by the stock of domestic capital net of the values of foreign debt and FPI, \( W = K_d - v_p e_p - PZ. \) Based on the agent’s portfolio decision, his wealth evolves as follows:

\[ dW = \left[ r_d(K_d - e_p) - r^*_t(PZ) + a_dL_d + a_fL_f - C \right] dt \\
+ \left[ f(K_d - e_p)dy_d - (PZ)dp \right] + \left[ \eta K_f + v_s e_s + (PZ) \right] ds, \]

where \( ds = \begin{cases} 0 & \text{with probability } 1 - \lambda dt, \\ 1 & \text{with probability } \lambda dt. \end{cases} \]

The term \( ds \) in equation (7) captures the change of the state from a normal time to a crisis situation in which the domestic economy defaults on all types of foreign capital inflows.
On the deterministic side, the evolution of domestic wealth in a given time interval \( dt \) is determined by the return on domestic capital, \( r_d K_d \), and labour income, \( \Sigma \alpha_i L_i \), less consumption, \( C \), the FPI investor’s claim, \( r_{dep} \), and the interest payment on foreign debt, \( r_c (PZ) \). We shall assume that both the domestic and foreign agents consume only deterministic amounts for smoothing purposes. On the stochastic side, the change in wealth during the normal time depends on the stochastic changes in output and the relative price. Upon default, the home country expropriates the entire portion of domestic capital financed by foreign debt and FPI, as well as an \( \eta \) fraction of FDI capital.

The assumption of a partial default on FDI is, as in Albuquerque (2003), based on the idea that FDI capital is harder to expropriate since the foreign control combines specific knowledge or operational skills that cannot be easily replicated by the domestic agent.

The foreign wealth, defined as \( W_f = K_f + v_p e_p + PZ \), evolves as follows:

\[
\begin{align*}
  dW_f &= \left[ r_f K_f + r_f e_f + r_c (PZ) + r_f W_f - C_f \right] dt \\
  &+ \left[ K_f f^f dy_f + e_f f^f dy_f + (PZ) dp \right] - \left[ \eta K_f + v_p e_p + (PZ) \right] ds.
\end{align*}
\]

Having their resources invested in the domestic economy, the foreign agents’ wealth grows based on the deterministic and stochastic returns from FDI, FPI, international lending, and a safe return, \( r_{ff} \), proportional to their wealth. In addition, the foreign agents experience a sudden drop in wealth if a default occurs.

Taking the prices, returns, and the interest rate as given, both the domestic and the foreign agents maximise the present values of their lifetime expected utility subject to their wealth constraints. First, a domestic agent solves the following maximisation problem:

\[
\begin{align*}
  V(W,t) &= \max_{C,K_f,PZ,e_p} E_t \left[ \frac{1}{\gamma} \right. \\
  \text{s.t. } dW &= \left[ r_d (K_d - e_p) - r_s (PZ) + a_d L_d + a_s L_f - C \right] dt \\
  &+ \left[ (K_d - e_p) f^d dy_d - (PZ) dp \right] + \left[ \eta K_f + v_p e_p + (PZ) \right] ds \\
  d\bar{W} &= \left[ r_f (\bar{K}_d - \bar{e}_p) - r_f (\bar{PZ}) + a_d L_d + a_f L_f - \bar{C} \right] dt \\
  &+ \left[ (\bar{K}_d - \bar{e}_p) f^d dy_d - (\bar{PZ}) dp \right] + \left[ \eta \bar{K}_f + v_p \bar{e}_p + (\bar{PZ}) \right] ds \\
  W &= K_d - v_p e_p - (PZ) \\
  K_d &= K_d + K_f \\
  K_f &= e_p \\
  \bar{PZ} &= 0.
\end{align*}
\]

The first and the third constraints are straightforward from the earlier discussions. The second constraint is obtained after taking averages of all choice variables. Since the agents behave as price takers, they also take the evolution of average wealth (denoted by an upper bar, \( \bar{W} \) and \( \bar{W}_f \)) into consideration when they make portfolio decisions. We shall assume representative agents for both countries in the discussions that follow so that all variables will be equal to the average values in equilibrium. The fourth constraint defines the composition of aggregate capital. The fifth constraint gives a restriction on the volume of FPI, which cannot exceed the stock of domestic capital, and the sixth constraint rules out the possibility of the domestic economy being a creditor.
3 Equilibrium

In this section, we derive the equilibrium portfolio shares of the both agents, growth of domestic wealth and its volatility. After that, we shall discuss some properties of the equilibrium. Postulating a time-separable value function, \( V(W, \bar{W}, t) = X(W, \bar{W})e^{-\beta t} \) we obtain the following first order conditions for the domestic agent with respect to consumption, domestic capital, foreign debt, and FPI respectively:

\[
C_{t+1}^{r+1} = X_{w}
\]

\[
\rho_i X_{w} - \rho_i = -X_{Ww} T_{j} \sigma_{j}^{2}(K_{d} - e_{p})
\]

\[
r_{j} X_{w} - \rho_i = X_{Ww} \sigma_{j}^{2}(PZ) + \lambda X_{ud}^{d}
\]

\[
r_{j} X_{w} - \nu_{p} \rho_i = -X_{Ww} T_{j} \sigma_{j}^{2}(K_{d} - e_{p}) + \lambda \nu_{p} X_{ud}^{d} - \rho_i.
\]

\( \rho_i \) (i = 1, 2, ..., 5) is the multiplier associated with the ith constraint in the domestic agent’s maximisation problem (similarly, \( \rho_i^* \) for the foreign agent). Condition (10) shows that the marginal utility of consumption must be equal to the marginal utility of wealth. Conditions (11) and (12) are asset pricing relationships on domestic investment and foreign debt, respectively. Postulating a value function of the form, \( X(W, \bar{W}) = \xi W^{r+1} - \bar{W}^{r} \) and considering \( W = \bar{W} \) in equilibrium, we can rewrite conditions (11) as \( r_{j} - \rho_i / \xi W^{r+1} = (1 - \gamma) \text{cov}(dW/W, dv_{j}) \), which relates the risk-adjusted return from domestic investment to the marginal utility of wealth with respect to the additional investment. Analogously, condition (12) turns into \( r_{j} - \rho_i / \xi W^{r+1} = (1 - \gamma) \text{cov}(dW/W, dp) + (C/C_{t+1}^{r+1})^{-}\gamma \). Notice that the revised expression for condition (12) includes an additional risk-adjustment term \( (C/C_{t+1}^{r+1})^{-}\gamma \), which accounts for the consumption foregone when the domestic economy defaults on foreign debt relative to the consumption after default. Combined together, both conditions yield an arbitrage condition between domestic investment and foreign debt, which in turn yields the domestic agent’s optimal portfolio shares of domestic investment and foreign debt. Similar to conditions (11) and (12), condition (13) determines how much FPI the domestic agent is willing to host.

The foreign agent’s first order conditions are obtained in an analogous manner:

\[
C_{f_{t+1}}^{r+1} = X_{w_{f}}
\]

\[
r_{j} X_{w_{f}} - \rho_{i}^{*} = -X_{W_{w_{f}}} f\sigma_{j}^{2} K_{d} + \lambda \eta X_{ud}^{d}
\]

\[
r_{j} X_{w_{f}} - \rho_{i}^{*} = -X_{W_{w_{f}}} \sigma_{j}^{2}(PZ) + \lambda X_{ud}^{d}
\]

\[
r_{j} X_{w_{f}} - \nu_{p} \rho_i = -X_{W_{w_{f}}} f\sigma_{j}^{2} e_{p} + \lambda \nu_{p} X_{ud}^{d} + \rho_i^{*}.
\]

The resulting equilibrium is a balanced growth path along which all real variables grow at the same stochastic rate. In such an equilibrium, the agents’ portfolio choices out of their wealth will be constant. Combining the first order conditions (10)~(13) and the wealth constraints, we obtain optimal portfolio shares of wealth in domestic capital and foreign debt as follows:
The optimal portfolio shares (18) and (19) are determined by three components. In each portfolio decision, return-interest differential in the first term determines the speculative component, the relative size of volatility determines the hedging component, and the third component reflects the role of FPI in domestic investment and foreign borrowing decisions. Greater return-interest differential encourages the home country’s physical capital accumulation through foreign borrowing, whereas higher domestic production volatility makes foreign debt riskier, thereby discouraging it. With the risk of default, the foreign agent charges a constant risk premium \( \lambda \). However, with the possibility of a sudden jump in wealth, the effective return-interest differential, \( r_d - r_z + \lambda (C / C^d)^{-\gamma} \), depends on the normal-to-default time consumption ratio as well. On the one hand, higher probability of default lowers the effective rate of interest. However, on the other hand, the increase in domestic wealth due to expropriation lowers the marginal utility of wealth (and consumption) after default, generating an opposite effect on the effective return-interest differential. The term \( \lambda (C / C^d)^{-\gamma} \) is derived from \( \lambda(W / W^d)^{-\gamma} \), which measures the expected marginal utility of defaulted foreign debt, relative to the marginal utility of foreign-debt-financed investment in normal times. Since the equilibrium consumption-wealth ratio and the normal-to-default time wealth ratio are constant in equilibrium, the effective return-interest rate is non-time varying. The third components in portfolio shares (18) and (19) show that FPI encourages both the domestic investment and foreign borrowing. In the limiting case of \( \epsilon_p \to 0 \), equations (18) and (19) are exactly the same expression as in the model with foreign debt only (Chung and Turnovsky, 2008). In contrast to the volume of FPI, the price of the equity has a positive effect on domestic investment, while it negatively affects foreign borrowing. As we will see later in this section, the equity of the domestic firm sells at a discount as long as the probability of default is positive. Intuitively, if the foreign agent’s claim on domestic output sells at a higher discount, the domestic agent has less incentive to host FPI and rely more on foreign debt, vice versa.

The foreign agent’s portfolio decision is determined analogously

\[
\begin{align*}
\left\{ K_d \right\}_d &= \frac{r_d - r_z + \lambda(C / C^d)^{-\gamma}}{(1 - \gamma)(f^2 \sigma_d^2 + \sigma_p^2)} + \frac{\sigma_p^2}{f^2 \sigma_d^2 + \sigma_p^2} + \frac{f^2 \sigma_d^2 + \sigma_p^2}{f^2 \sigma_d^2 + \sigma_p^2} \left( e_p \right) \\
\left\{ PZ \right\}_d &= \frac{r_d - r_z + \lambda(C / C^d)^{-\gamma}}{(1 - \gamma)(f^2 \sigma_d^2 + \sigma_p^2)} - \frac{f^2 \sigma_d^2 + \sigma_p^2}{f^2 \sigma_d^2 + \sigma_p^2} \left( e_p \right).
\end{align*}
\]

For the foreign agent, FPI substitutes both FDI and foreign lending, and the degree of substitution is proportional to the hedging component of the portfolio shares as well as the price of the equity. Along with the probability of default and normal-to-default time consumption ratio, the separability of FDI capital determines the effective return-interest
differential in the foreign agent’s portfolio choice. The separability parameter \( \eta \) affects the effective return-interest differential in two ways. As a direct effect, higher \( \eta \), in other words more expropriation of FDI capital upon default, reduces the differential, hence discourages FDI and encourages international lending. However, more expropriation of FDI capital decreases the equilibrium consumption after default, hence increases the consumption ratio \( (C_f/C_d)^{-\gamma} \), indirectly increasing the effective return-interest differential.

\[
e_p = \max \left\{ 0, \left( \frac{W_f}{W + W_f} \right) K_d + \psi \left( \frac{\sigma_p^2}{\sigma^2} \right) (PZ) \right\} \quad (22)
\]

\[
\psi_p = \frac{p C^{1-\gamma}}{\rho C^{1-\gamma} + \lambda (C/C^d)^{-\gamma}} = \frac{r_d - (1-\gamma) f^2 \sigma_p^2}{r_i + (1-\gamma) \sigma_p^2} \frac{(K_d - e_p)}{W} \cdot (PZ/W). \quad (23)
\]

The foreign agent determines FPI, the foreign-owned fraction of domestic capital, based on the foreign share of world wealth, taking into consideration the incompleteness in the capital market. Note that, without the second term, equation (22) turns into the expression for a perfect risk sharing in a complete capital market. The second term in equation (22) reflects the determinants of FPI due to the market incompleteness such as the probability of default and the discounted price of equity. The equity price, \( \psi_p \), is determined by equation (23). Without the risk of default, the equity price is always equal to unity. However, if \( \lambda > 0 \), each unit of equity sells at a discount, and the price discount increases as \( \lambda \) or \( (C/C^d)^{-\gamma} \) goes up.

Once the equilibrium portfolio shares are determined, we can derive the equilibrium consumption-wealth ratio, \( C/W \), mean growth rate of wealth, \( \psi \), and its volatility, \( \sigma^2_w \) as expressed in equations (24)–(26), respectively.

\[
\frac{C}{W} = \frac{1}{1-\gamma} \left[ \beta + \lambda \left( 1 - \left( \frac{C}{C^d} \right)^{1-\gamma} \right) - \gamma \left( r_d \left( \frac{K_d - e_p}{W} \right) - r_i \left( \frac{PZ}{W} \right) \right) \right] + \frac{\alpha f}{\rho C^{1-\gamma} + \lambda (C/C^d)^{-\gamma}} \frac{\gamma}{2} \sigma^2_w \quad (24)
\]

\[
\psi = \frac{1}{1-\gamma} \left[ r_d \left( \frac{K_d - e_p}{W} \right) - r_i \left( \frac{PZ}{W} \right) \right] - \beta - \lambda \left( 1 - \left( \frac{C}{C^d} \right)^{1-\gamma} \right) - \frac{\gamma}{2} \sigma^2_w \quad (25)
\]

\[
\sigma^2_w = \sigma^2_{w_e} + \sigma^2_{w_p},
\]

where \( \sigma^2_{w_e} = \left( \frac{K_d - e_p}{W} \right)^2 f^2 \sigma^2_p + \left( \frac{PZ}{W} \right)^2 \sigma^2_p \), and

\[
\sigma^2_{w_p} = \lambda \left( \frac{\eta K_f + v e_p + (PZ)}{W} \right)^2.
\]
Volatility of the growth of wealth, $\sigma_w^2$, in equation (26) measures the equilibrium variance along the balanced growth path. It consists of the volatility generated by Brownian-motion uncertainties and the one generated by discrete jumps in the wealth process. The former type, $\sigma_w^2$-volatility is generated by the domestic output and relative price volatilities and the domestic agent’s portfolio decision, while the latter type, $\sigma_w^2$-volatility is driven by the probability of default as well as the composition of foreign capital inflows. Depending on the domestic agent’s coefficient of risk aversion, the first type of volatility would affect the consumption-wealth ratio in either direction. A highly risk-averse domestic agent ($\gamma < 0$) reduces his consumption when the growth path of his wealth becomes more volatile. In contrast, the increase in $\sigma_w^2$-volatility lets him accumulate his capital faster. However, $\sigma_w^2$-volatility, driven by the possibility of default, does not directly influence the domestic agent’s consumption-wealth ratio or the average growth of his wealth. The possibility of default affects his consumption and growth of wealth directly through the probability of default, $\lambda$, and indirectly through the relative consumption in normal-to-default time, $(C/C^d)^{-\gamma}$. On the one hand, higher probability of default makes the domestic agent care less about saving. On the other hand, it discourages his consumption by increasing the effective return-interest differential by $\lambda(C/C^d)^{-\gamma}$. The same factors affect the mean growth rate of wealth in the opposite directions.

4 Calibration

In this section, we turn to numerical analysis. Table 1 summarises the parameter values used in calibration. Preference and technology parameters all take standard values. $\gamma$ in the utility function is set at $-1.5$, implying that the coefficient of relative risk aversion is 2.5. Production technology parameter $f = 0.3$ implies an approximate capital-output ratio of 3.33. Factor share of capital, $1 - \alpha$, is 0.4. We let the separability of FDI capital to vary from 0 to 1. At $\eta = 0$, the home country cannot gain any control over FDI capital even after default, while $\eta = 1$ means a complete expropriation. The probability of default $\lambda$ is set to vary from 0 to 0.2, implying no default at 0, and at 0.2, once-in-five-year default frequency on average. Ranges of the volatility parameters come from the data. Standard deviation of the annual per capita GDP growth rate during 1972–2006 period ranges from 1.5% to 3.3%, based on the country-income levels. For individual countries, there was a wider variation in the annual per capita GDP growth. Both the domestic and FDI output volatilities vary from 1% to 12%, with an increment of 1%. Considering the fact that production in developing countries tends to be more volatile than FDI production, we set the benchmark FDI output volatility at 2%, which is the standard deviation of OECD output growth, and the one for domestic output volatility at 6%, the median of our volatility parameter range. To proxy the relative price volatility, we calculate the standard deviation of real exchange rates during 1972–2006 period, which ranges approximately from 9% to 30% for 133 countries. We use the range of 2–24% in our calibration, with 2% increments. The benchmark for the relative price volatility is 8%. In the resulting equilibrium, implied debt-GNP ratio is 51.9%. Regarding the composition of foreign capital flows, the equilibrium share of foreign debt ranges approximately from 49% to 52.4%, FDI ranges from 1.3% to 50% (mostly in 22–34% range), and FPI ranges from 0% to 46% (mostly in 0–16% range) in our
calibration. These results are reasonably consistent with estimates from the actual data. For instance, Kraay et al. (2004) reports average values of net foreign debt, FDI liabilities and equity liabilities during 1990s as 70.8%, 27.4%, and 1.9%, respectively. Lane and Milesi-Ferretti (2001) reports averages of net foreign debt, FDI liabilities and equity liabilities of 45%, 45%, and 10%, respectively, over the same period.

Table 1  List of parameters

| Preference          | $\beta = 0.04$; $\gamma = -1.5$ |
| Production          | $f = 0.3; f' = 0.3$ |
|                     | $\delta = 0.02$; $\delta' = 0.04$ |
| Interest rate, bond | $r^* = 0.045$ |
| Relative price depreciation rate | $\epsilon = 0.01$ |
| Probability of default | $\lambda = 0~0.2$ |
| Volatilities        | $\sigma_p = 0.01~0.12$ (with 0.01 increment) |
|                     | $\sigma_f = 0.02~0.24$ (with 0.02 increment) |

Figure 1 shows the effects of volatility shocks on each agent’s portfolio decisions. Each panel on the left displays the domestic agent’s response and the one on the right shows the foreign agent’s. One can interpret the latter as the response of the composition in foreign capital inflows to the domestic economy. Figure 1(a) shows the effects of domestic output volatility shock. Other volatility values are set at the benchmark levels described above. As the domestic production becomes more volatile, the share of wealth in domestic investment (denoted by $w_d$) fluctuates along with the changes in FPI share ($w_p$) and gradually declines. As the only endogenous price variable in our model, $v_p$ is the main driving force of the changes in FPI share. Thus, the value of FPI in domestic wealth, $v_p e_p/W$, is more volatile than the share of its quantity, $e_p/W$. Compared to FPI, the share of foreign debt ($w_z$) in domestic wealth is relatively stable. In the composition of foreign capital flows, the share of foreign debt ($w_z^*$) is around 50%. As the domestic production becomes more volatile, most of the fluctuations in FPI ($w_p^*$) are replaced by changes in the share of FDI ($w_f^*$).

A similar pattern is observed in Figure 1(b), where the relative price volatility changes. Increasing relative price volatility makes foreign debt riskier. Therefore, international lending is mildly discouraged and FDI replaces the gap in the composition of foreign capital inflows. As foreign debt and FPI declines, domestic investment is also mildly discouraged.

Responses to FDI output volatility shock is much more stable and uniform than the previous two cases. Figure 1(c) shows that domestic wealth allocation is hardly affected. Domestic investment mildly increases due to the substitution of riskier FDI by foreign debt. FPI remains stable. A similar pattern is found in the composition of foreign capital inflows. As FDI production becomes riskier, it is replaced by a slight increase in the share of foreign debt.
Figure 1  Effects of volatility shocks on portfolio decisions: (a) effects of domestic output volatility shock; (b) effects of relative price volatility shock and (c) effects of FDI output volatility shock.
Figure 2 compares how volatility shocks can change the mean growth rate of wealth and its volatility, which are determined by the agent’s portfolio decision. Responses of the mean growth rates displayed in Figure 2(a) are consistent with our findings from equation (25), Figure 1 and Figure 2(b)–(d). From equation (25), we can see the volatility of wealth driven by Brownian-motion uncertainty accelerates the mean growth of wealth, as well as by the agent’s portfolio choice. When $\sigma_d$ or $\sigma_f$ increases, the mean growth rate of wealth, accompanied by variations in portfolio choice, rises with wide fluctuations. In contrast, when $\sigma_f$ increases, the mean growth rate mildly and steadily increases since the shock encourages domestic investment as foreign debt replaces FDI. We can observe similar patterns in the volatility of growth of wealth. Figure 2(b)–(d) illustrate how each volatility shock affects the volatility of growth. Consider the Brownian-motion generated volatility, $\sigma_w$ (denoted by $sw_1$), first. A positive volatility shock in $\sigma_d$ or $\sigma_p$ immediately destabilises the growth of wealth. In addition to that, there is an indirect, stabilising effect as the agent adjusts his portfolio shares. Depending on the relative size of these two effects, Brownian-motion volatility of wealth could be either destabilised or stabilised (Turnovsky and Chattopadhyay, 2003). In our calibration, the direct effect almost uniformly dominates the indirect effect, thereby steadily increasing $\sigma_w$. However, since FDI decision does not directly enter the domestic agent’s portfolio decision, increasing $\sigma_f$ hardly affects the volatility of domestic wealth. In contrast, the volatility of wealth generated by discrete jumps, $\sigma_p$ (denoted by $sw_2$), is not as sensitive to the volatility shocks as the Brownian-motion-generated volatility of wealth. Its response is more sensitive and significant to changes in the probability of default and the separability of FDI capital.

Figure 3 shows the effect of the change in the probability of default on the agents’ portfolio decisions and the volatility of wealth. As the probability of default increases, the domestic agent increases foreign borrowing slightly. In the very low range (probability less than 1%), higher probability of default decreases the share of foreign debt in the composition of foreign capital inflows. However, as the probability of default goes up further, its share slightly increases. Regarding the volatility of wealth, it is evident from equation (26) that an increase in $\lambda$ directly affects the average distance of jump and increases $\sigma_p$-volatility dramatically.

In Figure 4, we see the responses of portfolio decisions to the changing degrees of separability of FDI capital. As discussed in the previous section, higher $\eta$ implies that the domestic economy can expropriate a larger fraction of FDI capital upon default, and $\eta$ affects the foreign agent’s portfolio decision in two opposite directions. On the one hand it directly discourages FDI (encourages international lending) by reducing the effective return-interest differential through $(1-\eta)$ term. On the other hand, as $\eta$ increases, the indirect effect of the normal-to-default time consumption ratio $(C_I/C^*_I)^{\gamma-\eta}$ increases exponentially, thereby increasing the share of FDI, as shown in the right panel of Figure 4. Similar to the case of default probability change, an increase in $\eta$ causes a bigger jump in domestic wealth upon default, and directly affects $\sigma_p$-volatility.
Figure 2  Effects of volatility shocks on the mean growth of wealth and its volatility: (a) mean growth of wealth; (b) domestic output volatility shock; (c) relative price volatility shock and (d) FDI output volatility shock

Figure 3  Effects of default probability on portfolio decisions and volatility of wealth
**Figure 3** Effects of default probability on portfolio decisions and volatility of wealth (continued)

![Graph showing volatility of domestic wealth over probability of default](image)

**Figure 4** Effects of separability of FDI capital on portfolio decisions and volatility of wealth

(a) Domestic Wealth Allocation
(b) Foreign Capital Inflows
(c) Volatility of Domestic Wealth

![Graphs showing portfolio shares and separability](image)
5 Concluding remarks

In this paper, we investigate the effects of volatility shocks and other structural changes on the agent’s portfolio choice, growth of domestic wealth and its volatility in a small open economy. When the host country is subject to the risk of default, foreign investors seek for compensation. Depending on the types of foreign capital, such compensation takes different forms – default premium for foreign debt, discount price for FPI and non-separability for FDI. With the risk of default and potential jumps in the wealth process, the foreign lenders charge a risk premium, $\lambda$, equal to the mean rate of default. Although the returns to capital and the default premium on foreign debt are constant, the effective return-interest differential is not constant. It depends not only on the probability of default but also on the ratio of normal-to-default time consumption, due to the marginal contribution of investment in normal times relative to that after default.

The domestic agent’s investment and borrowing decision is complemented by FPI, while FDI and international lending decision by the foreign agent is substituted by it. The discount for the domestic equity also depends on the same determinants, the probability of default and normal-to-default time consumption ratio.

When there is a volatility shock in domestic output or in the relative price, the portfolio share of foreign debt remains relatively stable compared to other foreign investment decisions. In the composition of foreign capital inflows, most of the fluctuations of FPI, due to the changes in equity price, are offset by the fluctuations of FDI in the opposite directions. In contrast, a FDI output volatility shock causes only a mild substitution between foreign debt and FDI. Since the domestic output volatility or the relative price volatility incurs more significant fluctuations and adjustments in the agent’s portfolio decisions, it creates more fluctuation and acceleration in the average growth rate of domestic wealth. Such movements in the mean growth rate are due to the changes in the volatility of wealth generated by Brownian-motion uncertainties in output and relative price. Since the FDI volatility shock causes only mild and uniform adjustments in the agents’ portfolio decisions, the mean growth rate gradually and uniformly increases at a slower pace without incurring significant changes in the volatility of wealth.

Both the probability of default and separability of FDI capital affect the agents’ portfolio decisions in two directions. Increasing default probability directly widens the effective return-interest differential, thereby encouraging domestic investment and foreign borrowing. As an indirect effect, it discourages the investment and borrowing by altering the ratio of normal-to-default time consumption. In the composition of foreign capital inflows, the direct effect encourages (the indirect effect discourages) FDI and discourages (encourages) foreign lending. Moreover, higher probability of default increases the average distance of jump in the wealth process, making the growth of wealth more volatile. Increasing the separability of FDI capital brings similar effects on the portfolio decisions and the volatility of wealth.

Although the model does not explicitly analyses the government sector, our calibration results draw useful policy implications. First, the growth path of domestic wealth responds more sensitively to the volatility shocks in domestic output and relative price than to the FDI output shocks, which calls for more active stabilisation policy in such events. One possible policy instrument is to absorb stochastic changes in production more, via higher taxes on the same source. Higher taxes on uncertain changes in output, on the one hand, directly stabilises the growth path of domestic wealth. On the other
hand, higher taxes, hence greater absorption of uncertain domestic output encourage domestic investment via stabilising the production process, which in turn positively contribute to the labour income and welfare. Second, if the home country government were not to impose time-varying fiscal policy rules such as variable tax rates and variable absorption rates, its balanced-budget policy is not to impose taxes on foreign debt while compensating the gap between its expenditure and capital income tax by consumption or lump-sum taxes: for example, see Turnovsky (1999). However, if the home country experiences a default-probability shock or hosts a highly separable FDI, its government may use taxes on foreign debt as an effective tool to stabilise the Poisson volatility of growth in domestic wealth.

There are two issues to be addressed regarding the model. First, we assume a common probability of default that is constant and exogenously determined. However, it is not highly likely that the host country makes an economic decision to default on all foreign capital inflows at the same time. Furthermore, although our model introduces a default-risk premium to the interest rate on foreign debt, the domestic borrower still faces a perfectly elastic foreign debt supply curve despite sufficient empirical evidence of upward-sloping foreign debt supply curves. Therefore, endogenising the risk of default in each type of foreign capital will be a natural step to enrich the implications of the model. Second, our distinction between FDI and FPI is inconsistent to the definition in the national accounts. Moreover, in recent years, merger and acquisition has been growing in its share among FDI inflows to developing countries, obscuring the distinction between green field FDI and FDI through equity purchases. Incorporating these aspects into the model will provide more room to explore for the future research.

References


Notes

1 In national account, more than 10% share holding is considered as FDI. Our definition of FDI and FPI differ from the definition of national account in order to distinguish the green field FDI and foreign equity.

2 With the AK technology and the given process of the international bond price, $v_p$ is the only jump variable that guarantees a stable balanced growth path equilibrium in the world economy.

3 Detailed solution for each agent’s maximisation problem is delegated in the Appendix A and B.

4 With the definition of FDI in our model, the foreign agent’s FDI decision has no influence of the domestic agent’s portfolio decision. See Chung (2008).

5 World Development Indicators categorises the income groups into low income, lower-middle income, middle income, upper-middle income, high income, high income (OECD) and high income (non-OECD). World Development Indicators Online, The World Bank.


7 Global Development Finance categorises countries with debt-GNI ratio between 48% and 80% as moderately indebted. The average debt-GDP ratios during 1980–2005 period were 41.9 in low income countries, 55.8% in middle income countries and 33.6% in upper-middle income countries.

8 Note that the difference in the increments (1% for $\sigma_d$ and $\sigma_f$, 2% for $\sigma_p$) for each volatility shock is ignored in Figure 2(a).

9 It is obvious from equation (26). For a formal and detailed discussion, see Turnovsky and Chattopadhyay (2003).

10 For detailed discussions on the effect of various taxes in a similar setting, see Turnovsky (1999).

Bibliography


Appendix A: Equilibrium portfolio shares

In this appendix, we derive the equilibrium portfolio shares and the price of the equity. Let \( X \) and \( X' \) be the value functions of the domestic agent in normal times and after default, respectively. The Bellman equation for the price-taking domestic agent’s maximisation problem (9) is as follows:

\[
0 = \frac{1}{\gamma} C^\gamma - \beta X(W, \bar{W})e^{-\beta t} + X_w e^{-\beta t} [r_g (K_g - e_g) - r_f (PZ) + (a_d L_d + a_f L_f) - C] \\
+ \frac{1}{2} X_{ww} e^{-\beta t} [f^2 (K_g - e_g)^2 \sigma^2 + (PZ)^2 \sigma^2] \\
+ \frac{1}{2} X_{wW} e^{-\beta t} [f^2 (\bar{K}_g - \bar{e}_g)^2 \sigma^2 + (P\bar{Z})^2 \sigma^2] \\
+ X_{ww} e^{-\beta t} [f^2 (\bar{K}_g - \bar{e}_g) \sigma^2 + (P\bar{Z}) \sigma^2] \\
+ \lambda (W - K_g + PZ + v_p e_p + \eta K_f) - X(W) \\
+ \rho [W - K_g + PZ + v_p e_p] + \rho [K_d - K_d - K_f] \\
+ \rho [K_g - e_g] + \rho [PZ].
\]

(A1)

Postulating a value function of the form \( X(W, \bar{W}) = \xi W^{\gamma - \phi} \bar{W}^\phi \), we obtain

\[
X_w = (\gamma - \phi) \xi W^{\gamma - \phi - 1} \bar{W}^\phi.
\]

(A2)

And subsequently,

\[
X_{wW} = \phi X_{ww} W \bar{W}, \quad X_{ww} = (\gamma - \phi - 1) X_w W.
\]

(A3)

Using the fact that all real variables become equal to their averages in equilibrium, we obtain the domestic agent’s first order conditions (10)–(13), and the foreign agent’s first order conditions (14)–(17). Plugging equation (10) into equations (11)–(13), and combining equations (11), (12) and the domestic wealth constraint \( W = K_d - v p e_p - PZ \), we obtain the domestic equilibrium portfolio shares (18) and (19). The foreign
equilibrium portfolio shares (20) and (21) are obtained in an analogous manner by combining equations (14)–(16) and the foreign wealth constraint \( W_f = K_f + v_p e_p + PZ \). For the equilibrium quantity of FPI, subtract equation (17) from (13), and subtract equation (16) from (12). Then plug the latter into the former expression, which turns into equation (22). Finally, the price of the equity is a combination of equations (11)–(13).

Appendix B: Consumption-wealth ratio, growth rate and its volatility

In this section, we derive the domestic agent’s consumption-wealth ratio, mean growth rate of wealth and its volatility in equations (24)–(26), respectively. Let \( dW_r = dW - \lambda [\eta K_r + v_r e_r + PZ] \), and differentiate the domestic agent’s Bellman equation (A1) with respect to \( W \).

\[
\begin{align*}
0 &= Cz^{\gamma-1} C_w - \beta X_w + \psi X_w W + X_{ww} E_t \left( \frac{dW_w}{dt} \right) + X_{w\pi} E_t \left( \frac{dW_{\pi}}{dt} \right) \\
&\quad + X_{w\pi} \sigma_w^2 W + X_{w\pi} \sigma_w \bar{\pi} + \frac{1}{2} X_{ww\pi} E_t \left( \frac{dW_{\pi}}{dt} \right) + \frac{1}{2} X_{ww\pi} E_t \left( \frac{dW_{\pi}}{dt} \right) \\
&= X_w C_w - \beta X_w + X_w \left( \frac{\Delta}{W} - \frac{C}{W} \right) W + X_{ww} \left( \frac{\Delta}{W} - \frac{C}{W} \right) W \\
&\quad + X_{w\pi} \left( \frac{\bar{\pi}}{W} - \bar{C} \right) W + X_{w\pi} \sigma_w^2 W + X_{w\pi} \sigma_w \bar{\pi} + \frac{1}{2} X_{ww\pi} \sigma_w^2 W \\
&\quad + \frac{1}{2} X_{ww\pi} \sigma_w \bar{\pi} + X_{w\pi} \sigma_w \bar{\pi} W \bar{\pi},
\end{align*}
\]

where \( \Delta = r_d (K_r - e_p) - r_f (PZ) + (a_r L_r + a_r L_r) \) and \( W^d = K_d + \eta K_d \), which is domestic wealth after default. Using the results in equation (A3), equation (B1) becomes

\[
0 = X_w \left[ -\beta + \gamma \left( \frac{\Delta}{W} + (1 - \gamma) \frac{C}{W} + \lambda \left( \gamma \frac{C}{W} \right)^{-1} - 1 - \frac{\gamma(1 - \gamma)}{2} \sigma_w^2 \right) \right].
\]

Rearranging terms, we obtain the equilibrium consumption-wealth ratio in equation (24). From equation (24), it is straightforward to obtain the mean growth rate of wealth

\[
\psi = \left[ r_d \left( \frac{K_d - e_p}{W} \right) - r_f \left( \frac{PZ}{W} \right) + a_r L_r + a_r L_r \right] - \frac{C}{W}.
\]

For \( \sigma_{w} \), consider the Poisson uncertainty in the wealth process.

\[
dW_{\rho} = (\eta K_r + v_r e_r + PZ) ds = (K_d + \eta K_d - W) ds
\]

\[
= (W^d - W) ds = (W^d / W - 1) W \ ds
\]
Foreign capital inflows

\[ E_t \, dW_t = \lambda(W^d - W)dt = \lambda(W^d / W - 1)dt \tag{B5} \]

\[ E_t \, dW^2_t = \lambda(W^d / W - 1)^2 W^2 \, dt. \tag{B6} \]

Equations (B5) and (B6) yields \( \text{Var}(dW_t) = \lambda(W^d / W - 1)^2 W^2 \, dt \), which implies the expression in equation (26).