Foreign Debt, Foreign Direct Investment and Volatility

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Abstract

We investigate how foreign debt and foreign direct investment (FDI) affect the growth and welfare of a stochastically growing small open economy. First, we find that foreign debt influences the growth of domestic wealth by lowering the cost of capital, while FDI affects the country’s welfare by providing an additional source of permanent income. Second, a decline in domestic investment may improve domestic welfare as FDI replaces the gap. Even when the welfare deteriorates, its magnitude is mitigated, leaving more room for discretionary fiscal policy. Third, a fiscal policy aimed to stabilize domestic output fluctuations needs to be conducted not to crowd out the welfare benefit of FDI too much. Fourth, an economy with both types of foreign capital experiences wider welfare swings by external volatility shocks than the one with foreign debt alone, while the welfare effects from domestic volatility shocks are mitigated. The welfare effects of fiscal shocks are much smaller with both types of foreign capital. Lastly, the first-best labor income tax covers the government absorption by the labor’s share of total output, and the capital income tax covers the rest. Investment is penalized or subsidized depending on the social marginal cost-gain differential.

Keywords: Foreign direct investment, Foreign debt, Inelastic debt supply; Volatility

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1. Introduction

Along with the rapid increase in international capital flows and several episodes of financial crises in emerging markets during the 1990s, the issue of foreign capital flows and volatility has drawn much attention from many researchers. Among numerous topics in this issue, our focus is on the roles of foreign debt and foreign direct investment (FDI) on the host country’s wealth allocation and welfare in the face of volatility and fiscal policy shocks. Once a developing economy opens up its capital market, domestic firms obtain an easier access to international credits at a lower cost, and foreign investors enter the domestic capital market seeking for higher capital returns.

The basic framework of this paper is an extension of a stochastically growing small open economy model with inelastic foreign debt supply, employed in Turnovsky and Chattopadhyay (2003). The concept of inelastic foreign debt supply arises in several specifications and contexts. Focusing on the default risk on sovereign debt, upward sloping debt supply curves are: presented as a debt repayment function with lender imposed credit ceilings [Eaton and Gersovitz (1981)]; derived from the tradeoff between default and penalties [Grossman and van Huyck (1988), and Bulow and Rogoff (1989)]; introduced to guarantee stationarity in small open economy growth models [Mendoza and Uribe (2001), Senhadji (2003), and Turnovsky and Chattopadhyay (2003)]; or applied in modeling the episodes of financial crises [Rodrik and Velasco (1999), and Eicher, Turnovsky and Walz (2001)].

FDI, the second source of foreign capital flows in our model, also has been analyzed extensively in the literature, but the discussions were centered on the positive effects of FDI on the growth of recipient economy via technology transfer [Romer (1993)]. Its empirical evidence is mixed. Bosworth and Collins (1999) and Khawar (2005) report positive effects, Carkovic and Levine (2002) find no evidence, and some others find positive effects depending on the degree of complementarity and substitutability between FDI and domestic investment [de Mello (1999)], or a minimum threshold stock of human capital [Borensztein, De Gregorio and Lee, 1998]. In our model, where the returns to FDI
capital are completely remitted, the gain from FDI for the recipient country is realized as an increase in the domestic workers’ labor income from enhanced productivity of labor. Instead of dwelling on the FDI-growth debate, we set our primary focus on the relationship between volatility/policy shocks and the pattern of foreign capital inflows. In doing so, we analyze how the domestic and the foreign residents respond to the shocks, and see how the growth of wealth and welfare of the recipient economy are determined as a consequence of their optimal portfolio decisions.

The main findings of our model are: (i) foreign debt and FDI affect the recipient country’s growth and welfare through different channels. Foreign debt accelerates the growth by lowering the cost of capital while FDI improves the country’s welfare by providing an additional income source as well as the labor productivity spillover; (ii) a decline in domestic investment may improve domestic welfare as FDI replaces the gap. Even when the welfare deteriorates, its magnitude is mitigated, leaving more room for discretionary fiscal policy; (iii) a fiscal policy aimed to stabilize domestic output fluctuations needs to be conducted not to crowd out the welfare benefit of FDI too much; (iv) compared to a country with foreign debt alone, the one with both types of foreign capital experiences a wider welfare swing by an external volatility shock, while the welfare effect from a domestic volatility shock is mitigated. The welfare effect of a fiscal shock becomes much smaller with both types of foreign capital; and (v) the first-best labor income tax covers the government absorption by the labor’s share of total output, and the capital income tax covers the rest. Investment is penalized or subsidized depending on the social marginal cost-gain differential. Stochastic fluctuations in output must be taxed by the same fraction as the government absorption for stabilization purposes.

This paper is organized as follows. In section 2, we setup a model for our analysis. Equilibria for both the domestic and the foreign economies are characterized in section 3. After discussing simple comparative statics in section 4, we report the results from numerical analysis in section 5. Section 6 discusses the first-best tax structure and section 7 concludes our discussion.
2. The Model

There are two sources of externality inherent in the model. First, the aggregate stock of capital, regardless of the nationality of ownership, exerts a positive externality in production via spillover in the productivity of labor. The second source of externality comes from inelastic supply of foreign debt for the domestic borrowers. We shall assume that individual agents consider themselves unable to affect the world prices so that the individuals’ ignorance of the increasing marginal cost of foreign debt may lead to a suboptimal level of foreign borrowing in the aggregate level.

The world consists of two economies, the home and the foreign. The home country is a small open developing economy inhabited by \( L \) identical, infinitely-lived agents; the foreign country, or the rest of the world, is a large open economy with its own identical, infinitely-lived agents. There are \( N \) domestic firms and \( N^* \) FDI firms in the home country. Domestic firms are fully owned and operated by the domestic agents and FDI firms are fully owned and operated by the foreign agents.\(^1\) Combining home country labor with capital, both the domestic and FDI firms produce a single final product, which can be either consumed or invested without any cost of adjustment. The domestic firms finance its investment in part by domestic wealth and in part by foreign borrowing, while the FDI firms finance their capital purely by foreign wealth. Although both types of firms employ identical production technology, they face idiosyncratic shocks. Production functions of the domestic and the FDI firms are

\[
dY_i = \alpha(L_iK^A)\eta K_i^{1-\eta} (dt + dy_i) \quad (i = 1,\ldots,N) \tag{1.a}
\]
\[
dY_j^* = \alpha(L_j^*K^{A*})\eta K_j^{1-\eta*} (dt + dy_j^*) \quad (j = 1,\ldots,N^*) \tag{1.b}
\]

\(^1\) Our definition of FDI is different from the one by the IMF’s Balance of Payments Manual, 5\(^{th}\) edition (BPM5), which describes FDI as foreign acquisition of at least 10% ordinary shares or voting power of an enterprise abroad. We assume a full foreign ownership because, following the BPM5, it is difficult to tell FDI in the model from foreign portfolio investment (FPI), which generally has a shorter duration than FDI. Furthermore, the share of FPI is negligibly small in the composition of foreign capital inflows to developing countries. Kraay, Loayza, Servén and Ventura (2004) reports net foreign equity liabilities of 1.9%, compared to FDI’s 27.4% during 1990s. Lane and Milesi-Ferretti (2001) reports 10% and 45%, respectively, over the same period. Also it requires an introduction of an additional price variable, the relative price of the foreigners’ claims on the flow of domestic output. It tends to make the model analytically intractable when we analyze fiscal policy implications. A model in this direction, see Kraay et al. (2004).
where \( K^A = \sum_{i=1}^N K_i + \sum_{j=1}^N K_j \). Equations (1) show that a change in the production of final output in a given time interval, \( dt \), comprises a deterministic component and a stochastic component, both of which are governed by a Cobb-Douglas production technology. Combined with the aggregate capital stock, \( K^A \), domestic labor hired by a firm \( i \), \( L_i \) (\( L_j^* \) for a foreign firm), is put into the production process as an efficiency unit along with the physical capital, \( K_i \) (again, \( K_j^* \) for foreign). \( dy_i \sim N(0, \sigma^2_i dt) \) and \( dy_j^* \sim N(0, \sigma^2_i^* dt) \) represent, respectively, the stochastic components of output for the domestic and FDI firms and are assumed to be uncorrelated for simplicity. This production technology exhibits diminishing marginal returns to private capital, while it shows constant marginal returns at the aggregate level. We denote the variables owned or employed by FDI firms by the superscript *.

Each firm, prior to the realization of stochastic shocks, sets the wage deterministically by the marginal product of labor. The wage in the domestic firms, \( a_i dt \), and the one in the FDI firms, \( a_j^* dt \) can be expressed as follows:

\[
a_i = \left. \frac{\partial F_i}{\partial L_i} \right|_{L_A} = \eta f\left( \frac{K^A}{L^A} \right)^{1-\eta} \left( \frac{K_i}{L_i} \right); \quad a_j^* = \left. \frac{\partial F_j^*}{\partial L_j^*} \right|_{L_A^*} = \eta f\left( \frac{K^A}{L^A} \right)^{1-\eta} \left( \frac{K_j^*}{L_j^*} \right) \quad (2)
\]

where aggregate labor, \( L_A = \sum_i L_i + \sum_j L_j^* \), and \( f = \alpha L_A^{\eta} \). If factors move freely within a country, these wages will be equalized, and it ensures the capital-labor ratios in all firms equal the aggregate capital-labor ratio, \( K^A / L^A \), i.e., \( a_i = a_j^* = \eta f \left( K^A / L^A \right) \), which eventually determines labor income in the home country. In the discussions that follow, we drop the subscripts for individuals, \( i \) and \( j \), by assuming a representative agent in each economy.

Returns from domestic investment and FDI are determined as residuals of the final output after the payments for wages and depreciation, \( \delta \) (\( \delta^* \) for the FDI firms).

\[
dR_i = (dY - \delta dt - adt) / K = r_i dt + du_i = [(1 - \eta) f - \delta] dt + fdy \quad (3.a)
\]
The domestic agent’s debt financing is made by issuing internationally traded bonds. Its relative price in terms of the final output is denoted by $P$ and follows a geometric Brownian motion process:

$$dP = \varepsilon dt + dp, \quad dp \sim N(0, \sigma^2_p dt)$$

(4)

For a given volume of foreign debt, $Z$, the relative price determines the value of foreign debt, $PZ$. Each bond is a perpetuity for which the borrower continuously pays its interest over an infinite period of time. To focus on the effect of foreign capital inflows to a developing country, we shall confine our discussions to the case of a debtor nation. Unlike the foreign agents, the domestic borrowers are subject to the risk of default due to insufficiently endowed wealth as well as uncertainties in domestic production and the international bond price. To compensate for this risk, foreign lenders charge a premium, which is increasing in the size of foreign debt. We adopt a version of inelastic foreign debt supply, frequently employed in the literature. The interest rate on the risky foreign borrowing is defined as follows:

$$dR_z = r^* dt + dp = r^* dt + \xi \left( \frac{PZ}{W} \right) dt + dp,$$

(5)

where $r^*$ is the risk-free world interest rate. $\xi(\cdot)$ is the interest premium for the risk of default, and assumed to be an exponentially increasing function of debt-wealth ratio of the form, $\exp\{x \cdot (PZ / W)\} - 1$, where $x (> 0)$ is the curvature parameter, as originally introduced in Turnovsky and Chattopadhyay (2003). With the risk of default, the mean interest rate on the domestic investors' borrowing, $r_z$, will not be equal to that of the foreign investors' default-risk free rate, $r_z^* = r^* + \varepsilon$.

We assume that, the third agent, domestic government purely consumes away the final products from both the domestic and FDI firms by predetermined fractions. Government expenditure, $dG$, follows the rule

\[ dG = (gfK + g^* fK^*)dt + (g' fKdy + g'' fK^* dy^*) \]  

where \( g \) and \( g' \) denote the government absorption rates on the deterministic and the stochastic components of domestic output, whereas \( g^* \) and \( g'' \) are the fractions on FDI output. The government collects taxes from all types of income-generating activities in the home country to finance its expenditure. Without loss of generality, we ignore consumption tax since it does not affect the agents’ portfolio decisions, which are the main focus of this paper.\(^3\)

\[ dT = (\tau_x r_k dt + \tau_y dy)K + (\tau_x r'_k dt + \tau'_y dp)(PZ) + \tau_x (aL + a^* L^*) dt \]  

for the deterministic components, and the condition for the stochastic components,

\[ \tau'_y dy^* = \tau_x (PZ) dp . \]  

If the government were to use non-time-varying policy rule and to balance the budget regardless of the agent’s equilibrium portfolio choices, the tax rates must satisfy the following conditions:

\[ g = (1 - \eta)\tau_x + \eta \tau_y; \quad g^* = (1 - \eta)\tau_x^* + \eta \tau_y^*; \quad \tau_x = 0 \]

\[ g'_x = \tau'_y; \quad g'' = \tau''_x; \quad \tau'_y = 0 \]  

\(^3\) If we assume utility-generating government expenditure, consumption tax may influence the social optimum portfolio choice. See Turnovsky (1999).

\(^4\) \( \tau \) and \( \tau' \) (and \( \tau'' \)) are not necessarily equal because, in many countries, FDI firms receive various types of tax benefits as part of the host countries’ efforts to attract foreign capital.
The first three conditions state that the deterministic government consumption on domestic output must be financed by taxes on the deterministic income sources from the domestic firm, without penalizing the interest payment. The same rule applies to its consumption on FDI output. The next three conditions state that, in order to stabilize the fluctuations in output, the stochastic capital return tax rate on each firm must be set equal to the rate of stochastic government absorption on each output.

Given the returns, interest and tax rates, each agent decides how much to invest, borrow (or save) and consume out of his wealth, defined as the stock of domestic capital less foreign debt,

\[ W = K - PZ \ (W^* = K^* - PZ^* \text{ for the foreign}). \]

In addition, two world capital market clearing conditions follow, i.e., \( K^+ = K + K^* \), and \((PZ) + (PZ^*) = 0\). The first condition states that the stock of aggregate capital in the home country is the sum of domestically owned capital and the foreign-owned one. The second condition is the world credit market clearing condition.\(^5\)

Accumulation of domestic wealth is determined by the sum of capital return and labor income less consumption, interest and tax payments.

\[
dW = \left[ (1 - \tau_i)r_i K - r_z (PZ) + (1 - \tau_z)(aL + a^*L^*) - C \right] dt + \left[ (1 - \tau_i) fK dy - (PZ) dp \right] \tag{12.a}
\]

The foreign agent, as a creditor, accumulates his wealth from the FDI return, interest income and a fixed income stream \((dM^*)\) from his own country less consumption and payments for wages and taxes.

\[
dW^* = \left[ (1 - \tau_i^*)r^*_i K^* - r_z^* (PZ^*) + m^* - C^* \right] dt + \left[ (1 - \tau_i^*) fK^* dy^* - (PZ^*) dp \right] \tag{12.b}
\]

where \(dM^* = m^* W^* dt\) (\(m^*\) is a positive constant).\(^6\) Given the wealth constraints (12), each agent decides how to allocate his wealth among investment, foreign borrowing (or lending) and consumption in order to maximize his intertemporal utility. Both agents' investment and borrowing/lending decisions

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\(^5\) Due to the formulation of the wealth constraint, \((PZ) < 0\) when the agent is saving.

\(^6\) \(dM^*\) includes the foreigner's all possible income sources other than FDI return and interest income. Although its existence is not crucial for the implications of this model, it is required to satisfy the transversality condition for the equilibrium, which implies a positive \(C^*/W^*\) ratio.
determine the composition of aggregate capital in domestic capital \((n_k = K/K^4)\), FDI capital \((n_k^* = K^*/K^4)\) and foreign debt \((n_z = (PZ)/K^4)\), as well as domestic wealth \((W = K - (PZ) = (n_k - n_z)K^4)\). Then we can define implied portfolio shares of wealth in capital and foreign debt as follows:\(^7\)

\[
\begin{align*}
\omega_k &= \frac{K}{W} = \frac{n_k}{n_k - n_z}; \quad \omega_z = \frac{PZ}{W} = \frac{n_z}{n_k - n_z} \\
\omega_k^* &= \frac{K^*}{W^*} = \frac{n_k^*}{n_k^* - n_z}; \quad \omega_z^* = \frac{PZ^*}{W^*} = \frac{n_z^*}{n_k^* - n_z^*}
\end{align*}
\] (13.a) (13.b)

Throughout our discussion, the composition of aggregate capital will be denoted by \(n_i\)’s, whereas the portfolio allocation of wealth will be denoted by \(\omega_i\)’s. With the wealth constraint (12.a), we can write the domestic agent’s maximization problem as follows:

\[
V(W, \bar{W}, t) = \max_{C, n_k, n_z} \mathbb{E}_t \left[ \frac{1}{\gamma} \frac{1}{C(t)^{\gamma}} e^{-\alpha t} dt \right]
\]

s.t.

\[
\begin{align*}
\frac{dW}{W} &= \left[ r_k n_k / (n_k - n_z) - r_z n_z / (n_k - n_z) + (aL + a^* L^*) / W - C / W \right] dt \\
& \quad \left[ \left( n_k / (n_k - n_z) \right) f dy - \left( n_z / (n_k - n_z) \right) dp \right] - dT / W \\
\frac{d\bar{W}}{\bar{W}} &= \left[ r_k \bar{n}_k / (\bar{n}_k - \bar{n}_z) - r_z \bar{n}_z / (\bar{n}_k - \bar{n}_z) + (a\bar{L} + a^* \bar{L}^*) / \bar{W} - \bar{C} / \bar{W} \right] dt \\
& \quad \left[ \left( \bar{n}_k / (\bar{n}_k - \bar{n}_z) \right) f dy - \left( \bar{n}_z / (\bar{n}_k - \bar{n}_z) \right) dp \right] - d\bar{T} / \bar{W}
\end{align*}
\]

\(n_k - n_z + \bar{n}_k^* - \bar{n}_z^* = 1, \quad \text{and} \quad n_z + \bar{n}_z = 0\)

Solving this maximization problem, we assume that each agent considers the aggregate capital, average wealth, average portfolio shares and the mean interest rate on foreign debt, as well as the tax rates and his foreign counterpart’s optimal choice as given. Hence, he considers not only his own wealth, but also the average wealth of the home country, represented by the second wealth constraint. The upper bar denotes the average of each variable. Since all agents are identical, \(W = \bar{W}, \ n_k = \bar{n}_k, \ n_z = \bar{n}_z\), and \(C/W = \bar{C}/\bar{W}\) will hold in equilibrium. Once the composition of aggregate capital is chosen optimally, it

\(^7\) With foreign debt alone, \(n_z = \omega_z \ (i = k, z)\) and no distinction is necessary.
will determine the optimal portfolio allocation of wealth. The foreign agent solves a similar maximization problem with the wealth constraint (12.b).

3. Equilibrium

Solutions to the maximization problem (14) are derived in appendix A. With the first order condition (A.4) and the two capital market-clearing conditions, we obtain the shares of domestic capital and foreign debt in aggregate capital, \( \hat{n}_k \) and \( \hat{n}_z \), respectively:

\[
\hat{n}_k = \left(1 - \hat{n}_k^* + \hat{n}_l^* \right) \frac{\left[ (1 - \tau_k) r_k - r_r \right] + (1 - \gamma) \sigma^2_p}{\left[ (1 - \tau_k) \right]^2 f^2 \sigma^2_y + \sigma^2_p} \\
\hat{n}_z = \left(1 - \hat{n}_k^* + \hat{n}_l^* \right) \frac{\left[ (1 - \tau_z) r_z - r_r \right] - (1 - \gamma)(1 - \tau_k^*) \sigma^2_p}{\left[ (1 - \tau_z) \right]^2 f^2 \sigma^2_y + \sigma^2_p}
\]

Equations (15) are the domestic agent’s reaction functions. Taking the foreign agent’s decision on FDI and international lending as given, he makes his portfolio decision. The resulting portfolio shares, as part of the composition of aggregate capital, are determined by two factors: a speculative component, which is proportional to the after-tax return-interest differential; and a hedging component that depends on the relative volatility of the bond price (when investing) or that of the capital return (when borrowing). For the foreign agent, we obtain:

\[
\hat{n}_k^* = \left(1 - \hat{n}_k + \hat{n}_l \right) \frac{\left[ (1 - \tau_k^*) r_k - r_r \right] + (1 - \gamma) \sigma^2_p}{\left[ (1 - \tau_k) \right]^2 f^2 \sigma^2_y + \sigma^2_p} \\
\hat{n}_z^* = \left(1 - \hat{n}_k + \hat{n}_l \right) \frac{\left[ (1 - \tau_z^*) r_z - r_r \right] - (1 - \gamma)(1 - \tau_k^*) \sigma^2_p}{\left[ (1 - \tau_z) \right]^2 f^2 \sigma^2_y + \sigma^2_p}
\]

The composition of aggregate capital (\( \hat{n}_k \), \( \hat{n}_z \), \( \hat{n}_k^* \) and \( \hat{n}_z^* \)) in (15) and (16), with the international capital market clearing conditions, yields the optimal portfolio allocation of wealth, \( \hat{\omega}_k \) and \( \hat{\omega}_z \).
Using the equilibrium portfolio allocation of wealth together with the Bellman equation (B.1) and the wealth constraint (12.a), we solve for equilibrium consumption-wealth ratio, $\hat{C}/W$, mean growth rate of wealth, $\hat{\psi}$, and the volatility of growth in wealth, $\hat{\sigma}_w^2$ shown in (17.c), (17.d) and (17.e), respectively.\(^8\) Given the equilibrium consumption-wealth ratio in (17.c), we obtain the welfare of the economy, $\hat{\Omega}$ as in (17.f) from the Bellman equation (B.1).

\[
\hat{\psi} = \frac{1}{1 - \gamma} \left[ (1 - \tau_k) r_k \hat{\omega}_k - r_\omega \hat{\omega}_\omega \right] - \frac{\gamma}{2} \hat{\sigma}_w^2 \tag{17.d}
\]

\[
\hat{\sigma}_w^2 = (1 - \tau_k) \gamma \sigma^2_k \hat{\omega}_k + \sigma^2_{\rho} \hat{\omega}_\rho \tag{17.e}
\]

\[
\hat{\Omega} = \frac{(\frac{\hat{C}}{W})^\gamma W_0^\gamma}{\gamma \left[ \frac{\hat{C}}{W} - (\eta - \tau_k) f (\frac{\hat{C}}{W}) \right]}, \tag{17.f}
\]

where $r_k = (1 - \eta) f - \delta$ and $r_\omega = r_\omega + \xi(\omega_\omega)$. The set of equations (17) characterizes the domestic economy’s balanced growth path equilibrium where all economic variables grow on average at a common rate of $\hat{\psi}$. The equilibrium must satisfy the transversality condition, $\lim_{t \to -\infty} W^rf^{-\rho} = 0$, which turns into $C/W > 0$ and easy to verify.

Visual inspection of the portfolio shares (17.a) and (17.b) reveals that FDI and foreign debt play different roles in the determination of equilibrium in a decentralized economy. FDI capital initially generates a positive level effect through the productivity spillover, which enables ongoing growth of

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\(^8\) Formal derivation of $\frac{\hat{C}}{W}$ is provided in appendix B. Once $\frac{\hat{C}}{W}$ is obtained, plugging it into (12.a) yields $\hat{\psi}$.\(^8\)
domestic wealth. However, under the full foreign ownership with complete remittance of its returns, introduction of FDI contributes only to the home country’s welfare with the raised wage payment, and leaves the domestic agent’s portfolio allocation of wealth unaffected, thereby leaving the growth of domestic wealth and its volatility unaffected as well. In contrast, introducing foreign debt positively affects \( \hat{\omega} \) and \( \hat{\sigma} \) by lowering the cost of capital.\(^9\) Consequently, this adjustment in wealth allocation changes the mean growth rate of domestic wealth as well as its volatility. In fact, the separate roles of FDI and foreign debt are due to the setup of our model, which focuses on the growth of domestic wealth, not the growth of domestic production. Since we assume full foreign ownership of FDI capital and complete remittance of its capital returns, FDI’s contribution to the domestic economy is limited only to the productivity spillover effect, which is fully exploited by the increase in wages through a higher \( K'/L' \) ratio in equation (2). Therefore, once the productivity gain from FDI is fully exploited, the returns to capital (defined as residuals after wage and depreciation payments) on the balanced-growth path must be unaffected. Loosening up some of the assumptions, the model may generate growth effects by FDI. For instance, if we relax our assumption on FDI to allow foreigners’ equity holdings in domestic firms, the introduction of FDI may bring down the cost of capital as foreign debt does. However, as long as its capital returns are remitted to the foreign country, its contribution to the growth of domestic wealth would still be limited. A more plausible possibility is when we allow cross-correlation between domestic and FDI output. Then the introduction of FDI will affect the domestic agent’s portfolio allocation of wealth through the volatility channel, by changing the relative size of risk in (17.a) and (17.b).\(^10\) The equilibrium consumption-wealth ratio (17.c) shows that the effects of foreign capital on consumption and growth are realized not only by the size of widened return-interest differential, but also

\(^9\) Under mild conditions on the risk-free world interest rate and the interest premium function, it is straightforward to show that the mean interest rate on foreign debt, \( r_z \), is lower than its counterpart in autarky, \( r_z^a = (1- \tau) r_k - (1 - \gamma) f^2 \sigma^2 \). For a range of reasonable parameter values, calibration of the model shows lower \( r_z \) than \( r_z^a \).

\(^10\) The denominator of (17.a) and (17.b) changes to \( f^2 \sigma^2 + \sigma^2 + 2 f^2 \sigma_{y*} \), where \( \sigma_{y*} = \text{cov}(dy, dy*) \) and a nonlinear combination of the foreigner’s equity share and volatility terms will be added, which may not allow closed-form solutions as we have in (17.a) ~ (17.f).
by the volatility of wealth. The former, together with the additional labor income from FDI firms, positively affects the equilibrium consumption, whereas the latter may be either positive or negative, depending on the sign of $\gamma$. A highly risk averse agent with a coefficient of relative risk aversion greater than 1, i.e., $\gamma < 0$, will cut down his consumption when the growth path of his wealth becomes more volatile. Since such a consumption cut will be saved and invested for the uncertain future, the economy can grow faster on average as the wealth constraint (12.a) implies.\footnote{Ramey and Ramey (1995) show empirical evidence on the tradeoff between volatility and growth. For a positive correlation, see Kormendi and Meguire (1985). Imbs (2007) provides evidence on a positive correlation between sectoral volatility and growth and a negative correlation between macroeconomic volatility and growth.} In (17.f), there are two main determinants of the agent’s welfare: consumption-wealth ratio and labor income. A higher consumption-wealth ratio directly improves welfare by increasing utility. On the other hand, it takes away the resources for investment, which lowers the prospect of economic growth. The sign of the latter effect depends on the coefficient of relative risk aversion. For a highly risk averse agent, higher steady-state consumption matters more than higher growth. The welfare implication of labor income also depends on $\gamma$ through the growth channel, since it is ultimately determined by the capital-wealth ratio in both the domestic and the FDI firms.\footnote{From $d\Omega = \chi d(\gamma^*) / d(\gamma^*) - \Omega[ d(\gamma^*) - (\eta - \tau_s) d(\gamma^*) ] / [ \gamma^* - (\eta - \tau_s) f(\gamma^*) ]$, the first term is the direct channel and the second one is the indirect channel.}

For the foreign agent, let $\hat{\omega}_i^* = \hat{n}_i^* / (\hat{n}_i^* - \hat{n}_i^*)$ and $\hat{\omega}_z^* = \hat{n}_z^* / (\hat{n}_z^* - \hat{n}_z^*)$. Then with the reaction functions (16), we obtain the following equilibrium system:

\[
\hat{\omega}_k^* = \frac{\left[ (1 - \tau_s) r_k^* - r_s \right] + (1 - \gamma) \sigma_p^2}{(1 - \gamma) \left[ (1 - \tau_s)^2 f^2 \sigma_y^2 + \sigma_p^2 \right]} 
\]  \hspace{1cm} (18.a)

\[
\hat{\omega}_z^* = \frac{\left[ (1 - \tau_s) r_z^* - r_s \right] - (1 - \gamma)(1 - \tau_s)^2 f^2 \sigma_y^2}{(1 - \gamma) \left[ (1 - \tau_s)^2 f^2 \sigma_y^2 + \sigma_p^2 \right]} 
\]  \hspace{1cm} (18.b)

\[
\hat{C}^*_W = \frac{1}{1 - \gamma} \left[ \beta - \gamma \left\{ (1 - \tau_s)(r_k^* \hat{\omega}_k^* - r_z^* \hat{\omega}_z^*) \right\} \right] + m^* + \frac{\gamma}{2} \hat{\sigma}_W^2 
\]  \hspace{1cm} (18.c)
\[ \hat{\psi} = \frac{1}{\gamma} \left[ \left( 1 - \tau^*_k \right) r^*_k \hat{\omega}^*_k - r^*_z \hat{\omega}^*_z \right] - \beta \frac{\gamma}{2} \hat{\omega}^*_z \]  
\[ (18.d) \]

\[ \hat{\sigma}_{w^2}^2 = (1 - \tau^*_k)^2 \sigma^*_k \hat{\omega}^*_k + \sigma^*_z \hat{\omega}^*_z \]  
\[ (18.e) \]

\[ \Omega^* = \frac{(\hat{c}_w)^{\gamma} W_0^{\gamma}}{\gamma (\hat{c}_w + \eta f \hat{\omega}^*_k)} \]  
\[ (18.f) \]

where \( r^*_k = (1 - \eta) f - \delta^* \). As long as the after-tax return from FDI capital is higher than the default-risk free interest rate, \( r^*_z \), in order for the foreign agent to be a net creditor, \( \sigma_{y^2}^* \) should be sufficiently high to ensure \( \hat{\omega}^*_z < 0 \).\(^{13}\) Although the mean interest rate for the default-free foreign agent is a certainty equivalent \( r^*_z \), he considers \( r^*_z = r^*_z + \xi(\hat{\omega}^*_z) \) as the opportunity cost of FDI because the interest premium, \( \xi \), is the profit from financial intermediation as a creditor. The adjustment of the interest premium is the main factor that insures an equilibrium in the international financial market to rule out corner solutions.

4. Some comparative statics

From the general properties of the equilibrium in the previous section, we could see that the entire equilibrium system is affected by the agents’ portfolio decisions. Based on the returns and risks, each agent’s decision determines the composition of aggregate capital, \( \hat{n}_i \), and it implicitly determines the portfolio allocation of wealth, \( \hat{\omega}_i \) (\( i = k, z \)). The portfolio allocation of domestic wealth is of our particular interest because it is the crucial determinant of the mean and the variance of the growth of domestic wealth. In this section we will provide a brief sketch on two channels, \( \hat{\omega}_i \) and \( \hat{\sigma}_{w^2}^2 \), through which a volatility or fiscal shock generates growth effects.

Consider two output volatility shocks, an increase in \( \sigma_{y}^2 \) and an increase in \( \sigma_{y^2}^2 \). The former, a domestic output volatility shock affects the agent’s portfolio allocation of wealth as follows:

\(^{13}\) It is natural to assume substantially high \( \sigma_{y^2}^2 \) since FDI firms tend to have less information about the host economy. However, it does not necessarily require \( \sigma_{y^2}^2 \) to be greater than \( \sigma_{y}^2 \).
\[
\frac{\partial \hat{\omega}_k}{\partial \sigma_y^2} = \frac{\partial \hat{\omega}_z}{\partial \sigma_y^2} = -(1 - \tau'_k)^2 f^2 \times \frac{\hat{\omega}_k + \partial \xi / \partial \sigma_y^2}{\left[ (1 - \tau'_k)^2 f^2 \sigma_y^2 + \sigma_p^2 \right] + \partial \xi / \partial \hat{\omega}_k}
\]  

(19)

In addition to its direct effect, \(-(1 - \tau'_k)^2 f^2 \hat{\omega}_k / [(1 - \tau'_k)^2 f^2 \sigma_y^2 + \sigma_p^2]\), the shock shifts up the foreign debt supply curve by the distance of \(\partial \xi / \partial \sigma_y^2\) and, as a response, the domestic agent adjusts his portfolio by \(\partial \xi / \partial \hat{\omega}_k\). Depending on the portfolio adjustment effect on the interest premium, domestic capital out of wealth may increase or decrease. In our calibration, increasing domestic production risk uniformly discourages domestic investment. Once the shock generates such a portfolio reallocation effect, it in turn changes the volatility of the growth path as follows:

\[
\frac{\partial \hat{\sigma}_w^2}{\partial \sigma_y^2} = (1 - \tau'_k)^2 f^2 \hat{\omega}_k^2 + 2 \left[ (1 - \tau'_k)^2 f^2 \sigma_y^2 + \sigma_p^2 \right] \left( \hat{\omega}_k - \bar{\omega}_k \right) \left( \frac{\partial \hat{\omega}_k}{\partial \sigma_y^2} \right),
\]  

(20)

where \(\bar{\omega}_k = \sigma_p^2 / [(1 - \tau'_k)^2 f^2 \sigma_y^2 + \sigma_p^2]\) is the volatility-minimizing share of capital in domestic wealth.\(^{14}\)

The first term in equation (20) is the direct effect from the shock and it is always destabilizing. The second term is the indirect effect due to the domestic agent’s portfolio adjustment. It may stabilize or destabilize the growth path, depending on the relative size of \(\hat{\omega}_k\) to \(\bar{\omega}_k\) and the sign of portfolio adjustment. If the home country is a debtor, the sign of the secondary effect is always equal to the sign of his portfolio reallocation effect.

In contrast, a positive FDI output volatility shock causes disproportionate adjustments in the composition of aggregate capital, leaving the domestic portfolio allocation of wealth unchanged.\(^{15}\) It is not a surprising result under the assumptions of fully foreign-owned FDI and no correlation between the domestic and FDI output processes as discussed in the earlier section.

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\(^{14}\) We choose \(\hat{\omega}_k\) by setting the partial derivative of \(\sigma_y^2\) with respect to \(\omega_k\) at zero. In this case, however, the domestic agent is a creditor, not a debtor. See Turnovsky and Chattopadhyay (2003) for a detailed discussion.

\(^{15}\) From the definition of wealth allocation (14), \(\hat{\omega} \hat{\sigma} / \hat{\sigma} = (\hat{\omega} (\hat{\sigma} \hat{\sigma} / \hat{\sigma}) - \hat{\omega} (\hat{\sigma} \hat{\sigma} / \hat{\sigma}) / (\hat{\sigma} - \hat{\sigma}))\) for \(x = \sigma_y^2, \sigma_z^2, \tau, \tau', \tau', \tau', \tau', \tau' \) \((i = k, z)\). It is obvious that the adjustment in the composition of aggregate capital does not necessarily lead to a change in the allocation of domestic wealth.
5. Calibration

In this section we analyze the effects of volatility and fiscal policy shocks with the results from calibration. We begin by listing the parameter values used in our numerical analysis in Table 1.

*****Table 1 enters here*****

The parameter values in Table 1 are set for our benchmark equilibrium to provide a plausible approximation of the real world data for developing economies. Bold typefaced entries in Table 2 represent the equilibrium in our benchmark economy. In Table 2 (a), each cell reports the domestic agent’s portfolio decisions in terms of \( \hat{n}_k \) and \( \hat{n}_z \), the interest premium, the mean growth rate, volatility of wealth, labor income, consumption-wealth ratio and the deviation of welfare from the benchmark. Except for \( \hat{n}_k \) and \( \hat{n}_z \), all values are reported in percent.\(^{16}\) Also note that, being independent of FDI, his portfolio allocation of wealth, \( \hat{k}_d \) and \( \hat{z}_d \), are identical to those in an economy with foreign debt alone, which we report in Table 2 (b). Moving from the upper to the lower blocks in Table 2 (a), we track the response of the economy to the changes in FDI output volatility, \( \sigma_{y2}^2 \), while the movement from the left to the right blocks tracks the effects of the changes in the relative price volatility, \( \sigma_p^2 \). In each main block, there are three subdivisions that track the changes in the domestic output volatility, \( \sigma_{y2}^2 \).

In Table 2 (a), \( \hat{n}_k \) ranges from 53.3 to 99.8%, whereas \( \hat{n}_k^* \) ranges from 0.2 to 46.7%. During 1990 ~ 2006 period, the ratio of inward FDI in gross fixed capital formation ranges from 4.1% to 16.2% with an average 10.3% in developing economies.\(^{17}\) Our benchmark of 21.4% is a bit higher than what the data suggest, but individual country data show much wider variations. For example, Vietnam’s average is 23.1% with minimum 10.6% and maximum 49.2%, similar to our calibration result. Implied debt-GNP ratio (\( = \hat{n}_z / (f \cdot \hat{n}_k) \)) varies from 16% to 27.9% in our model, whereas its average ranges

\(^{16}\)The labor income and the consumption-wealth ratio are reported as their percentage in domestic wealth. The change in welfare is in percentage change from the benchmark equilibrium welfare level.

\(^{17}\)World Investment Report, UNCTAD (2007).
from 33 to 55.8% in developing countries over 1980 ~ 2005 period.\textsuperscript{18} Consumption-wealth ratio for the domestic economy is around 18.1 ~ 31.5%, implying that 60.3% ~ 105% of GNP is consumed.\textsuperscript{19}

****[Table 2 (a) and (b) enter here]****

5.1 Responses to volatility shocks

We find several common properties of the growth and welfare effects of a volatility shock. First, increasing output volatility discourages investment in the relevant capital, and the sign of its welfare effect depends on the level of relative price volatility. If foreign lending is too risky a substitute of FDI for the foreign agent, a domestic output volatility shock will attract enough FDI to generate a positive welfare effect, while an FDI volatility shock may not attract as much foreign borrowing to fill in for the loss of domestic investment, thereby lowering domestic welfare. Second, unlike the case of a domestic output volatility shock, an FDI volatility shock does not influence the mean and the variance of the growth path of domestic wealth since the portfolio allocation of domestic wealth remains the same. Third, given the volatility of FDI output, increasing volatility (either domestic output or the relative bond price) induces more FDI in the domestic economy.\textsuperscript{20} Therefore, introduction of FDI by offering an additional source of labor income, provides the domestic economy with a buffer against unfavorable volatility shocks. Fourth, compared to the economy with foreign debt alone, an economy with both types of foreign capital experiences a wider welfare swing by an external-source driven volatility shock. In contrast, the welfare effect generated by a domestic-source driven volatility shock is mitigated with both foreign capital inflows.

\textsuperscript{18} Sample countries were categorized in low, medium-low, medium, medium-high, and high income groups. World Development Indicators, World Bank (2006). International Financial Statistics, IMF (2007).

\textsuperscript{19} In order to satisfy the transversality condition and match the domestic consumption-wealth ratio, we set the fixed return $m^*$ in the foreign country at 0.15, which is close to the share of labor income in the domestic economy.

\textsuperscript{20} Empirical evidence on the relationship between FDI and volatility (particularly the real exchange rate volatility) are mixed. For a positive relationship, see Cushman (1988) and Goldberg and Kolstad (1995). For a negative one, see Bénassy-Quéré, Fontagné and Lahrèche-Révil (2001).
We start with an increase in domestic output volatility. Higher $\sigma^2_p$ discourages foreign borrowing as well as domestic investment (lower $\hat{n}_i$ and $\hat{n}_k$), which makes the foreign agent to switch his portfolio more toward FDI (increasing $\hat{n}_k^*$). If the low level of $\sigma^2_p$ makes foreign debt, $\hat{n}_i^*$, the predominant form of foreign capital inflows, the additional labor income from increasing FDI does not provide enough support for domestic consumption, thereby deteriorating welfare as in the case of a debt-only economy in Table 2 (b). However, if the initial $\hat{n}_i^*$ is large enough (for medium to high $\sigma^2_p$), the switch to more FDI will increase the labor income, consumption and welfare. Despite the decrease in domestic investment, the domestic wealth grows faster on average for three reasons: the greater return-interest differential, reduced consumption out of wealth, and the increase in the volatility of wealth.

In contrast, riskier FDI output (higher $\sigma^2_p$) changes the composition of aggregate capital in favor of domestic capital (higher $\hat{n}_i$ and lower $\hat{n}_k^*$). Furthermore, the size of such redistribution depends on the level of relative price volatility, $\sigma^2_p$. If the foreign agent considers international lending a risky substitute for FDI (due to higher $\sigma^2_p$), an increase in $\sigma^2_p$ may not lead to an increase in $\hat{n}_i^*$ in the same proportion as to the decline in $\hat{n}_i^*$, which leaves $\hat{\omega}_k$ and $\hat{\omega}_z$ unaltered. Since there is no portfolio reallocation in domestic wealth, the interest premium, growth of wealth and its volatility all remain unchanged in the home country. However, the decline in FDI decreases the domestic agent’s labor income, consumption, and welfare. The deterioration of welfare is more severe with a higher $\sigma^2_p$ because the foreign agent’s switch from FDI to international lending will be too limited to boost up domestic investment to provide enough cover for the welfare from decreasing FDI.

---

21 Without FDI, less investment means lower productivity of labor, hence lower labor income and consumption, which uniformly deteriorates welfare.
Finally, like a $\sigma^2$ shock, a positive volatility shock in the relative price of bonds (higher $\sigma^2$) discourages domestic investment and foreign borrowing, but to a much greater extent. Therefore, the direct destabilizing effect on the volatility of wealth is mitigated by a large indirect portfolio adjustment effect discussed in the previous section, and the mean growth rate increases by less than in the case of a $\sigma^2$ shock. Moreover, the decline in investment and foreign borrowing is replaced by a substantial increase in FDI, which increases the labor income and consumption to a greater extent, thereby improving domestic welfare much further. With foreign debt alone [Table 2 (b)], changes in the portfolio decision, growth of wealth and its volatility show very similar patterns. However, having no additional source of labor income, an increase in $\sigma^2$ uniformly deteriorates consumption and welfare of the debt-only economy.

*****[Table 3 (a) and (b) enters here]*****

5.2 Responses to tax rate changes

Table 3 (a) and (b) summarize the effects of fiscal policy shocks in our model and the debt-only model, respectively. First, whether domestic or foreign or both, raising the tax rate on any deterministic capital return discourages the relevant investment and deteriorates the domestic welfare. However, the nationality of capital matters on the directions and magnitudes of the tax rate change on consumption, growth of domestic wealth and its volatility. Taxing the deterministic returns to domestic capital discourages the incentive to invest, thereby increasing consumption out of wealth and slows down the growth, whereas taxing the deterministic FDI capital returns discourages consumption with no growth effect. Second, regardless of the nationality of capital, changing the tax rate on the deterministic capital returns affects the composition of aggregate capital, consumption and labor income to a greater extent and in the opposite directions than an equal change in the stochastic capital return tax would do. It is because a higher deterministic capital tax discourages the agent’s incentive to invest directly by
lowering the after-tax return, while its stochastic counterpart indirectly encourages investment by funding the government absorption of uncertain output fluctuations. Third, as a higher tax rate on the stochastic capital returns redistributes the composition of aggregate capital in favor of the relevant capital, such portfolio reallocation induces, although small in magnitude, a welfare effect that can be either positive or negative depending on its effect on consumption out of wealth. Taxing the stochastic return from domestic capital is very weakly welfare-deteriorating, while taxing its FDI counterpart is welfare-improving. However, in the debt-only economy, more absorption of output fluctuations is always stabilizing and welfare-improving. Fourth, whether deterministic or stochastic, changing the tax rate on the domestic capital returns generates a smaller welfare effect than changing the tax rate on the FDI returns. It is due to the fact that part of the fiscal shock on the domestic capital return is mitigated by the domestic agent’s portfolio reallocation of wealth, while the fiscal shock on the FDI counterpart only affects the labor income only, with no such reallocation effect. Lastly, in an economy with foreign debt only, an equal-sized fiscal shock (either deterministic or stochastic) generates a greater welfare-swing than in an economy with both types of foreign capital.

A higher tax rate on the deterministic return to domestic capital (higher $\tau_\delta$) discourages domestic investment and foreign borrowing (decreasing $\hat{\delta}_x$ and $\hat{\delta}_z$ as well as $\hat{\omega}_x$ and $\hat{\omega}_z$). Reduced investment slows down the growth of wealth and stabilizes its volatility at the same time. Furthermore, the decrease in domestic investment causes more consumption out of reduced wealth and partially replaced by higher FDI, which results in increasing labor income and consumption. Nevertheless, the domestic welfare very weakly deteriorates because the welfare effect of increasing consumption is dominated by the reduced-growth effects. We observe a similar wealth reallocation effect in the debt-only economy from Table 3 (b). However, domestic firms being the only source of income, decreased foreign borrowing and investment in the debt-only economy leads to more deterioration of domestic
welfare. It implies that the introduction of FDI would leave more room for discretionary fiscal policy with less impact on welfare.

Raising the tax rate on the deterministic FDI return (higher $\tau^*_x$) forces the foreign agent to switch from FDI to more international lending. It changes the composition of aggregate capital in favor of domestic investment. This lowers the shares of labor income and consumption out of domestic wealth, which deteriorates the welfare of domestic economy without affecting the growth and its volatility.

The effects of a higher tax rate on the stochastic capital returns (higher $\tau_x$ or $\tau^*_x$) are qualitatively identical to those from a negative output volatility shock. It is because, as discussed earlier, such a fiscal shock exerts a stabilizing effect on the growth of wealth through government consumption. More absorption of stochastic domestic production encourages domestic investment and foreign borrowing, while more absorption of stochastic FDI output attracts more FDI and discourages international lending in the composition of aggregate capital. In the former case, the positive growth effect from increasing domestic investment is dominated by the negative growth effect from stabilization, so its net effect on growth is mildly negative. Domestic welfare, although the magnitude is negligibly small, deteriorates due to three reasons, all of which crowds out private consumption: increasing investment; lowered labor income from discouraged FDI; and more government consumption. In contrast, the latter case does not induce such a portfolio reallocation effect in wealth and hence leaves the mean growth rate of wealth and its volatility unaffected. Furthermore, the increase in FDI leads to mildly increasing labor income and consumption, which result in a mild welfare improvement.

Tax on labor income does not influence portfolio choices of both the agents. The tax effect falls solely on each agent’s consumption decision, so a higher labor income tax rate deteriorates domestic welfare. With these findings we discuss the first-best tax structure of the decentralized economy in the following section.
6. First-best tax structure

The first-best policy for the home country government is to set the tax rates so that the outcome of each agent’s economic decision mimicks the equilibrium determined by a benevolent social planner in a centralized economy. We shall start our discussion on the optimal tax structure by comparing the equilibrium portfolio shares of the decentralized economy to the socially optimal allocation with arbitrarily given rates of government absorption, \( g \) and \( g' \), then proceed to the optimal choice of the absorption rates. In doing so, we assume that the primary focus of the social planner is to choose the absorption rates to stabilize the growth path of domestic wealth at given portfolio shares and private consumption.\(^2\) Since the home country is a small open economy, he considers the foreign wealth and the fraction of domestic wealth out of world wealth as given, and maximizes domestic welfare given in equation (15.a). However, unlike the representative agent, he takes into account the externality inherent in the upward-sloping debt supply curve, as well as productivity spillover in the labor market. The resulting equilibrium determines the following:

\[
\omega_{k,o} = \frac{(1-g)[(1-\eta)f - \delta] - (r_x + \xi') + \eta f}{(1-\gamma)[(1-g')^2 f^2 \sigma_y^2 + \sigma_p^2]} + \frac{\sigma_y^2}{(1-g')^2 f^2 \sigma_y^2 + \sigma_p^2} \quad (21.a)
\]

\[
\omega_{c,o} = \frac{(1-g)[(1-\eta)f - \delta] - (r_x + \xi') + \eta f}{(1-\gamma)[(1-g')^2 f^2 \sigma_y^2 + \sigma_p^2]} - \frac{(1-g')^2 f^2 \sigma_y^2}{(1-g')^2 f^2 \sigma_y^2 + \sigma_p^2} \quad (21.b)
\]

\[
\sigma_{w,o}^2 = (1-g')^2 f^2 \sigma_y^2 \omega_{k,o}^2 + \sigma_p^2 \omega_{z,o}^2 \quad (21.c)
\]

where the subscript \( o \) represents the social optimum. Since the social planner internalizes the external effects of foreign borrowing and productivity gain, the wealth allocation in (21.a) and (21.b) are different from (17.a) and (17.b) in two aspects. On the one hand, additional investment via additional

\(^2\) One reason is that, in our current setup, government spending is pure consumption. It only crowds out private expenditure without affecting the private agents’ consumption and investment decisions via consumption-externality or provision of productive infrastructure. For discussions in these directions in stochastic growth models, see Corsetti (1997) for a closed economy, and Turnovsky (1999) for an open economy with perfect capital mobility.
foreign debt raises the social cost of borrowing, which lowers the social return-interest differential by the marginal change in interest premium, $\xi'$. On the other hand, the knowledge spillover generated by an increase in investment, $\eta f$, positively adds to the social return-interest differential as a social gain. If the former exceeds the latter, the domestic agent in the decentralized economy overborrows (hence overinvests) than the social optimum, and in the opposite case underborrows (and underinvests).

Since the marginal increase in the premium is variable while the gain from the knowledge spillover is constant, they may not be always equal for a given set of tax and absorption rates. Thus, depending on the relative size of these social cost and gain, the equilibrium portfolio shares (17.a) and (17.b) in the decentralized economy may be either greater or smaller than the socially optimal portfolio shares (21.a) and (21.b). Setting (17.a) and (21.a) equal and plugging in $(1 - \eta)\tau_a + \eta \tau_s$ from the first condition of balanced-budget condition (11) for a given rate $g$, we obtain

$$\tau_s = \tau_a + \frac{\xi' - \eta f}{\eta r},$$

(22)

where $r = (1 - \eta)f - \delta$ as defined previously. Equation (22) shows that the first-best tax rates on capital return and wage may or may not be set equal depending on the gap between the social cost of additional borrowing and the social gain from additional investment. Consider the case where the marginal premium exceeds the productivity gain, i.e., $\xi' > \eta f$. In order to prevent overborrowing, the government sets the capital return tax rate higher than the labor income tax rate. The extra charge is the social marginal cost exceeding the social marginal gain, normalized by the before-tax private return on investment and the share of labor income out of total production. In the opposite case, investment is subsidized by a lower tax rate. If the social marginal cost and gain are equal, the first-best policy is to set $\tau_s = \tau_a = g$, a rule under which the capital income tax covers the government absorption by the share of capital income in total output $(1 - \eta)$ and the labor income tax covers the rest by the share of labor income $(\eta)$.
Now we turn to the choice of the socially optimal government purchases. Partial differentiation of $\sigma_w^2$ in (21.c) with respect to $g$ ($g'$ for the stochastic component) yields expressions for the optimal government absorption rates as follows:

$$g_o = \frac{(r_k + \eta f) - (r_c + \xi')}{(1 - \eta) f - \delta}$$

(23.a)

$$g'_o = 1$$

(23.b)

In (23.a), $r_k + \eta f$ is the social mean rate of return on investment at which the productivity spillover in the labor market is fully internalized, whereas $r_c + \xi'$ is the social mean rate of interest at which the upward-sloping nature of foreign debt supply is internalized. Thus, for a given private mean rate of return on investment, a wider social return-interest differential (hence likely underinvestment) calls for a greater absorption by the government to encourage domestic investment to the socially optimal level.

Now we can rewrite the first-best tax rule (22) as follows:

$$\tau_k = g_o + \frac{\xi' - \eta f}{\eta r_k}; \quad \tau_a = g_o$$

Under this rule, the labor income tax always covers the government absorption by the share of labor’s contribution to total output, and the capital income tax covers the rest. As discussed earlier in this section, if the marginal social cost exceeds the marginal social gain, investment must be penalized, and in the opposite case, subsidized.

In contrast, the variance-minimizing government absorption on the stochastic domestic production equals 1, implying that the government should increase its stochastic consumption during

---

23The expressions are $\frac{\partial \sigma_w^2}{\partial g} = -2r_1(\Delta \omega_{..} - \sigma^2)\sqrt{(1 + \gamma)\Delta + \xi^2 + \xi'^2} = 0$ for the deterministic component, and $\frac{\partial \sigma_w}{\partial g'} = -2(1-g')f'g'(1 - \gamma)\Delta \omega_{..}\sqrt{(1 - \gamma)\Delta + \xi^2 + \xi'^2}$ for the stochastic component, where $\Delta = (1 - g')f^2 \sigma^2 + \sigma^2$. 23
economic booms and decrease it during recessions by taxing all of the stochastic changes in output. In a more realistic case of $g' < 1$, the first-best tax rate on stochastic output will still be $\tau_k' = g'$.

7. Conclusion

In this paper we have analyzed the effects of volatility and fiscal policy shocks on the growth and welfare of a developing economy that hosts two types of foreign capital inflows, foreign debt and FDI. We develop an analytical model that characterizes a developing small open economy with a liberalized but imperfect capital market. Then, with the help of calibration, we present several features of the balanced-growth equilibrium in the face of volatility and fiscal policy shocks.

First, our model predicts that foreign debt and FDI affect the host country’s growth and welfare through different channels. Foreign debt accelerates the growth of domestic wealth by lowering the cost of physical capital even with an inelastic, positively sloped foreign debt supply curve. In contrast, FDI improves the host country’s welfare by raising the domestic labor’s permanent income.

Second, in analyzing the effects of volatility and fiscal policy shocks, we need to understand how the composition of aggregate capital and the allocation of wealth among domestic investment, foreign debt and FDI affect the equilibrium. Any types of shocks alter the composition of aggregate capital, which in turn influences the domestic agent’s labor income, consumption and welfare. However, as long as the shock leaves the domestic agent’s wealth allocation unaffected, there would be no further effects on the growth of domestic wealth and its volatility.

Third, with both foreign debt and FDI, a shock that discourages domestic investment does not necessarily deteriorate domestic welfare because FDI provides an additional source of permanent income. Furthermore, as the decline in domestic investment is accompanied by a decrease in foreign borrowing, FDI prevents the outflow of domestic wealth in the form of interest payment. For these reasons, even when the shock causes a negative welfare effect, its magnitude is relatively small to the
one in an economy with foreign debt alone. It provides the domestic government with more room for a discretionary fiscal policy than in an economy with foreign debt alone.

Fourth, with both types of foreign capital, an active fiscal policy aimed to stabilize unexpected domestic output fluctuations may crowd out FDI too much, deteriorating domestic welfare. The negative welfare effect may be negligibly small in magnitude, but care must be taken prior to using fiscal policy for stabilization purposes. On the contrary, an active fiscal policy to stabilize FDI output improves the domestic welfare. In terms of the magnitude, a fiscal shock on the domestic sources of income generates a smaller welfare effect than the one on the FDI sources.

Fifth, compared to the economy with foreign debt alone, the one with both types of foreign capital experiences a wider welfare swing by a volatility shock originating from external sources, while the welfare effects generated by domestic sources of volatility are mitigated. In the case of a fiscal shock, an economy with both types of foreign capital experiences a smaller welfare swing.

Lastly, the first-best tax policy is to set the labor income tax to cover the deterministic government absorption by the labor's share of total output and to cover the rest by the capital income tax. If the social marginal cost of foreign debt exceeds the social marginal gain from productivity spillover, capital income must be penalized proportional to the net marginal cost, and in the opposite case it must be subsidized proportional to the net marginal gain. Stochastic fluctuations in output must be taxed by the same fraction as the government absorption for stabilization purposes.
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<th>Parameters</th>
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<td>$\sigma_{\epsilon} = 0.08, 0.16, 0.24$</td>
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<td>Tax rates</td>
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Table 2 (a) Effects of volatility shocks

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<th>$\hat{n}_k$</th>
<th>$\psi$</th>
<th>$\hat{\sigma}_w^2$</th>
<th>$C/W$</th>
<th>$\hat{n}_y$</th>
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<td></td>
<td>0.1</td>
<td>0.972</td>
<td>2.544</td>
<td>4.830</td>
<td>18.088</td>
<td>0.752</td>
<td>2.548</td>
<td>4.849</td>
<td>21.574</td>
</tr>
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<td>0.2</td>
<td>0.972</td>
<td>2.544</td>
<td>4.830</td>
<td>18.088</td>
<td>0.752</td>
<td>2.548</td>
<td>4.849</td>
<td>21.574</td>
</tr>
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<td>0.2</td>
<td>0.05</td>
<td>0.994</td>
<td>2.433</td>
<td>2.512</td>
<td>18.059</td>
<td>0.827</td>
<td>2.444</td>
<td>2.647</td>
<td>20.327</td>
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<td>0.974</td>
<td>2.544</td>
<td>4.830</td>
<td>18.005</td>
<td>0.810</td>
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<td>4.849</td>
<td>20.432</td>
</tr>
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<td>0.2</td>
<td>0.974</td>
<td>2.544</td>
<td>4.830</td>
<td>18.005</td>
<td>0.810</td>
<td>2.548</td>
<td>4.849</td>
<td>20.432</td>
</tr>
</tbody>
</table>

Note:
1. $\hat{n}_k$ is obtained by calculating $1 - \hat{n}_k$.
2. $\hat{\omega}_k$ and $\hat{\omega}_y$ at each level of $\sigma_y$ in our model are identical to the entries in Table 2 (b).

Table 2 (b) Effects of volatility shocks (debt-only economy)

<table>
<thead>
<tr>
<th>$\sigma_f$</th>
<th>$\sigma_y$</th>
<th>$\hat{\omega}_k$</th>
<th>$\psi$</th>
<th>$\hat{\sigma}_w^2$</th>
<th>$C/W$</th>
<th>$\hat{\omega}_y$</th>
<th>$\xi$</th>
<th>wage</th>
<th>$\Delta\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.05</td>
<td>1.083</td>
<td>2.404</td>
<td>1.389</td>
<td>18.045</td>
<td>1.072</td>
<td>2.417</td>
<td>1.667</td>
<td>17.923</td>
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<td>0.1</td>
<td>1.080</td>
<td>2.433</td>
<td>2.512</td>
<td>17.985</td>
<td>1.069</td>
<td>2.444</td>
<td>2.647</td>
<td>17.868</td>
</tr>
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<td>1.067</td>
<td>2.544</td>
<td>4.830</td>
<td>17.745</td>
<td>1.058</td>
<td>2.548</td>
<td>4.849</td>
<td>17.647</td>
</tr>
<tr>
<td>0.1</td>
<td>0.05</td>
<td>1.080</td>
<td>2.433</td>
<td>2.512</td>
<td>17.985</td>
<td>1.069</td>
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<td>2.647</td>
<td>17.868</td>
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<td>1.080</td>
<td>2.433</td>
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<td>1.058</td>
<td>2.548</td>
<td>4.849</td>
<td>17.647</td>
</tr>
</tbody>
</table>

Note:
1. $\hat{\omega}_k$ and $\hat{\omega}_y$ at each level of $\sigma_y$ in our model are identical to the entries in Table 2 (b).
### Table 3 (a) Effects of tax rate change

<table>
<thead>
<tr>
<th>Tax Change in the Deterministic Return on Domestic Capital</th>
<th>( \hat{\omega}_k )</th>
<th>( \hat{\omega}^* )</th>
<th>( \hat{n}_k )</th>
<th>( \hat{n}_k^* )</th>
<th>Prem</th>
<th>( \hat{\psi} )</th>
<th>( \hat{\sigma}_w^2 )</th>
<th>( c/w )</th>
<th>wage</th>
<th>( \Delta\Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_k = 0.20 )</td>
<td>0.823</td>
<td>0.066</td>
<td>0.177</td>
<td>1.087</td>
<td>0.730</td>
<td>2.191</td>
<td>2.708</td>
<td>2.811</td>
<td>21.004</td>
<td>19.811</td>
</tr>
<tr>
<td>( \tau_k = 0.25 )</td>
<td>0.786</td>
<td>0.051</td>
<td>0.214</td>
<td>1.069</td>
<td>0.808</td>
<td>1.742</td>
<td>2.444</td>
<td>2.647</td>
<td>21.136</td>
<td>20.394</td>
</tr>
<tr>
<td>( \tau_k = 0.30 )</td>
<td>0.725</td>
<td>0.035</td>
<td>0.275</td>
<td>1.051</td>
<td>0.886</td>
<td>1.293</td>
<td>2.184</td>
<td>2.505</td>
<td>21.860</td>
<td>21.763</td>
</tr>
</tbody>
</table>

| Tax Change in the Stochastic Return on Domestic Capital |
|---------------------------------------------------------|----------------|
| \( \tau_k = 0.20 \)                                   | 0.785 | 0.050 | 0.215 | 1.069 | 0.810 | 1.729 | 2.449 | 2.789 | 21.150 | 20.419 | 0.007 |
| \( \tau_k = 0.25 \)                                   | 0.786 | 0.051 | 0.214 | 1.069 | 0.808 | 1.742 | 2.444 | 2.647 | 21.136 | 20.394 | 0.000 |
| \( \tau_k = 0.30 \)                                   | 0.788 | 0.051 | 0.212 | 1.070 | 0.806 | 1.754 | 2.440 | 2.507 | 21.123 | 20.371 | -0.005 |

| Tax Change in the Deterministic Return on FDI Capital |
|---------------------------------------------------------|----------------|
| \( \tau_k = 0.20 \)                                   | 0.665 | 0.043 | 0.335 | 1.069 | 0.886 | 1.742 | 2.444 | 2.647 | 23.923 | 24.109 | 13.810 |
| \( \tau_k = 0.25 \)                                   | 0.786 | 0.051 | 0.214 | 1.069 | 0.808 | 1.742 | 2.444 | 2.647 | 21.136 | 20.394 | 0.000 |
| \( \tau_k = 0.30 \)                                   | 0.852 | 0.055 | 0.148 | 1.069 | 0.730 | 1.742 | 2.444 | 2.647 | 19.964 | 18.831 | -7.085 |

| Tax Change in the Stochastic Return on FDI Capital |
|---------------------------------------------------------|----------------|
| \( \tau_k = 0.20 \)                                   | 0.789 | 0.051 | 0.211 | 1.069 | 0.806 | 1.742 | 2.444 | 2.647 | 21.091 | 20.334 | -0.255 |
| \( \tau_k = 0.25 \)                                   | 0.786 | 0.051 | 0.214 | 1.069 | 0.808 | 1.742 | 2.444 | 2.647 | 21.136 | 20.394 | 0.000 |
| \( \tau_k = 0.30 \)                                   | 0.784 | 0.051 | 0.216 | 1.069 | 0.810 | 1.742 | 2.444 | 2.647 | 21.179 | 20.451 | 0.244 |

| Tax Change in the Labor Income |
|--------------------------------|----------------|
| \( \tau_a = 0.20 \)                                   | 0.786 | 0.051 | 0.214 | 1.069 | 0.808 | 1.742 | 2.444 | 2.647 | 22.156 | 20.394 | 5.497 |
| \( \tau_a = 0.25 \)                                   | 0.786 | 0.051 | 0.214 | 1.069 | 0.808 | 1.742 | 2.444 | 2.647 | 21.136 | 20.394 | 0.000 |
| \( \tau_a = 0.30 \)                                   | 0.786 | 0.051 | 0.214 | 1.069 | 0.808 | 1.742 | 2.444 | 2.647 | 20.117 | 20.394 | -6.113 |

### Table 3 (b) Effects of tax rate change (debt-only economy)

| Tax Change in the Deterministic Capital Return (No FDI) |
|---------------------------------------------------------|----------------|
| \( \tau_k = 0.20 \)                                   | 1.087 | 0.087 | 2.191 | 2.708 | 2.811 | 18.370 | 16.300 | 8.063 |
| \( \tau_k = 0.25 \)                                   | 1.069 | 0.069 | 1.742 | 2.444 | 2.647 | 17.868 | 16.036 | 0.000 |
| \( \tau_k = 0.30 \)                                   | 1.051 | 0.051 | 1.293 | 2.184 | 2.505 | 17.366 | 15.771 | -9.148 |

| Tax Change in the Stochastic Capital Return (No FDI) |
|---------------------------------------------------------|----------------|
| \( \tau_k = 0.20 \)                                   | 1.069 | 0.069 | 1.729 | 2.449 | 2.789 | 17.858 | 16.028 | -0.142 |
| \( \tau_k = 0.25 \)                                   | 1.069 | 0.069 | 1.742 | 2.444 | 2.647 | 17.868 | 16.036 | 0.000 |
| \( \tau_k = 0.30 \)                                   | 1.070 | 0.070 | 1.754 | 2.440 | 2.507 | 17.877 | 16.044 | 0.132 |

| Tax Change in the Labor Income |
|--------------------------------|----------------|
| \( \tau_a = 0.20 \)                                   | 1.069 | 0.069 | 1.742 | 2.444 | 2.647 | 18.670 | 16.036 | 5.131 |
| \( \tau_a = 0.25 \)                                   | 1.069 | 0.069 | 1.742 | 2.444 | 2.647 | 17.868 | 16.036 | 0.000 |
| \( \tau_a = 0.30 \)                                   | 1.069 | 0.069 | 1.742 | 2.444 | 2.647 | 17.066 | 16.036 | -5.664 |

28
Appendix

A. Derivation of the equilibrium portfolio decisions

Here we derive the equilibrium for the domestic agent. We have two state variables, individual wealth \( W \) and average wealth \( \overline{W} \). The Stochastic Bellman equation has the following form:

\[
0 = \max_{C; \alpha, \gamma} \left\{ \frac{C^\gamma}{\gamma} e^{-\beta t} + L[V(W, \overline{W}, t)] \right\},
\]

subject to the two wealth constraints and the capital market clearing conditions. The term \( L[V(W, \overline{W}, t)] \) is the differential generator, which means the expected value of the change in the value function over an infinitesimally small period \( dt \), i.e., \( \lim_{dt \to 0} E(dV / dt) \). Therefore, it can be rewritten as

\[
L[V(W, \overline{W}, t)] = \lim_{dt \to 0} E \left( \frac{\partial V}{\partial t} + V_w \frac{dW}{dt} + V_{\overline{W}} \frac{d\overline{W}}{dt} + \frac{1}{2} V_{ww} \frac{dW^2}{dt} + \frac{1}{2} V_{\overline{W}\overline{W}} \frac{d\overline{W}^2}{dt} + V_{W\overline{W}} \frac{dW d\overline{W}}{dt} \right)
\]

We postulate a time separable value function \( V(W, \overline{W}, t) = X(W, \overline{W}) e^{-\beta t} \) and rewrite the stochastic Bellman equation as

\[
0 = \max_{C; \alpha, \gamma} \frac{C^\gamma}{\gamma} e^{-\beta t} + (X_t - \beta X) e^{-\beta t}
\]

\[+ X_w W e^{-\beta t} \left[(1 - \tau_i) \rho_i \omega_i - \rho_i \omega_i + (1 - \tau_z)(\omega_i + \theta \cdot \overline{\omega}_i) - C/W \right]
\]

\[+ X_{\overline{W}} \overline{W} e^{-\beta t} \left[(1 - \tau_i) \rho_i \overline{\omega}_i - \rho_i \overline{\omega}_i + (1 - \tau_z)(\overline{\omega}_i + \overline{\theta} \cdot \overline{\omega}_i) - C/\overline{W} \right]
\]

\[+ \frac{1}{2} \sigma_w^2 X_{ww} W^2 e^{-\beta t} + \frac{1}{2} \sigma_{\overline{W}}^2 X_{\overline{W}\overline{W}} \overline{W}^2 e^{-\beta t} + \sigma_{w\overline{W}} X_{w\overline{W}} W \overline{W} e^{-\beta t}
\]

\[+ \lambda e^{-\beta t} (1 - n_i + n_i - \overline{n}_i + \overline{n}_i) + \lambda e^{-\beta t} (n_i - \overline{n}_i),
\]

where \( r_i = (1 - \eta) f - \delta \), \( \sigma_w^2 = (1 - \tau_i) \omega_i \sigma^1 + \omega_i \sigma^2 \), \( \sigma_{\overline{W}}^2 = (1 - \tau_i) \overline{\omega}_i \sigma^1 + \overline{\omega}_i \sigma^2 \), \( \theta = W / W^* \), \( \overline{\theta} = \overline{W} / \overline{W}^* \), \( \omega_i = n_i / (n_i - n_i) \), \( \overline{\omega}_i = \overline{n}_i / (\overline{n}_i - \overline{n}_i) \), \( i = k, z \).

Recalling that \( W = \overline{W} \), \( n_i = \overline{n}_i \) and \( \omega_i = \overline{\omega}_i \) \( (i = k, z) \) hold in equilibrium, we obtain the following relationship between the two returns from the first order conditions:

\[
\left( [(1 - \eta) f - \delta] - r_i \right) dt = (1 - \gamma) \text{cov}(dW, (1 - \tau_i) fdy - dp)
\]

(A.4)
Equation (A.4) is the arbitrage condition between domestic investment and foreign borrowing. If the economy is on a balanced growth path, it will have a recurring equilibrium system. In such an equilibrium, stocks of domestic capital, FDI capital, and the value of international debt will grow at a common rate, which guarantees constant $C/W, n_i$ and $\omega_i$ ratios ($i = k, z$). Once a shock perturbs the equilibrium, a new equilibrium will be restored through a change in the mean interest rate, more specifically through an adjustment in the interest premium. From equation (A.4) and the two capital market clearing conditions, we obtain the shares of domestic capital and foreign debt both in aggregate capital and in domestic wealth as expressed in equations (15.a), (15.b) and (17.a), (17.b), respectively.

**B. Derivation of the equilibrium consumption-wealth ratio without government**

In this section, we derive the equilibrium $C/W$ ratio in (17.c). We can rewrite the stochastic Bellman equation (A.3) as follows:

$$0 = \frac{1}{\gamma} C^\gamma - \beta X + X_w \frac{E(dW)}{dt} + X_w \frac{E(d\bar{W})}{dt} + \frac{1}{2} X_{ww} \frac{E(dW^2)}{dt} + \frac{1}{2} X_{w\bar{w}} \frac{E(d\bar{W}^2)}{dt} + X_{w\bar{w}} \frac{E(dWd\bar{W})}{dt}$$

(B.1)

Using the first-order condition $C^\gamma = X_w(W, \bar{W})$, we can differentiate (B.1) with respect to $W$ and obtain

$$0 = C^\gamma X_w - \beta X_w + X_{ww} \frac{E(dW)}{dt} + X_w (r_\omega k - r_\omega k' + \eta f(\omega_k + \theta \omega_k') - C_w) + X_{w\bar{w}} \frac{E(d\bar{W})}{dt} + \frac{1}{2} X_{ww} \frac{E(dW^2)}{dt} + \frac{1}{2} X_{w\bar{w}} \frac{E(d\bar{W}^2)}{dt} + X_{w\bar{w}} \frac{E(dWd\bar{W})}{dt}$$

(B.2)

Now take the stochastic differential from $X_w = X_w(W, \bar{W})$:

$$dX_w = X_{ww} dW + X_{w\bar{w}} d\bar{W} + \frac{1}{2} X_{www} dW^2 + \frac{1}{2} X_{w\bar{w}} d\bar{W}^2 + X_{w\bar{w}} dWd\bar{W}$$

(B.3)

Taking expectations and dividing by $dt$, we get

$$\frac{E(dX_w)}{dt} = X_{ww} \frac{E(dW)}{dt} + X_{w\bar{w}} \frac{E(d\bar{W})}{dt} + \frac{1}{2} X_{www} \frac{E(dW^2)}{dt} + \frac{1}{2} X_{w\bar{w}} \frac{E(d\bar{W}^2)}{dt} + X_{w\bar{w}} \frac{E(dWd\bar{W})}{dt}$$

(B.4)

From (B.2),
\[ RHS \text{ of (B.4)} = -C^{r-1}C_w + \beta X_w - X_w (r \omega_k - r \omega_z + \eta f(\omega_k + \theta \omega^*_k) - C_w) \]
\[ -\sigma_w^2 W X_{ww} - \sigma_{w\sigma}^2 W X_{w\sigma} \]  
(B.5)

Using \( C^{r-1} = X_w(W, \widetilde{W}) \) again,

\[ X_w (r \omega_k - r \omega_z + \eta f(\omega_k + \theta \omega^*_k) - \beta) + \sigma_w^2 W X_{ww} + \sigma_{w\sigma}^2 W X_{w\sigma} + \frac{E(dX_w)}{dt} = 0 \]  
(B.6)

We postulate \( X = \delta W^\gamma \tilde{W}^\alpha \). Then we obtain

\[ X_w = \delta(\gamma - \phi) W^\gamma \tilde{W}^\alpha / W; \quad X_{w\sigma} = \delta \phi W^\gamma \tilde{W}^\alpha / \widetilde{W} \]
\[ X_{ww} = (\gamma - \phi - 1) X_w / W; \quad X_{w\sigma w} = (\phi - 1) X_{w\sigma} / \widetilde{W}; \quad X_{ww} = \phi X_w / \widetilde{W} \]

From (B.6),

\[ \frac{E(dX_w)}{dt} = \left[ \beta - (r \omega_k - r \omega_z) - \eta f(\omega_k + \theta \omega^*_k) \right] + (1 - \gamma + \phi) \sigma_w^2 - \phi \sigma_{w\sigma} \]  
(B.7)

Now we take stochastic differentials of \( C^{r-1} = X_w(W, \tilde{W}) \):

\[ dX_w = (\gamma - 1) C^{r-2} dC + \frac{1}{2} (\gamma - 1)(\gamma - 2) C^{r-3} dC^2 \]
\[ \frac{dX_w}{X_w} = (\gamma - 1) \frac{dC}{C} + \frac{1}{2} (\gamma - 1)(\gamma - 2) \left( \frac{dC}{C} \right)^2 \]

Taking expectations in the above equation,

\[ \frac{E(dX_w)}{X_w} = (\gamma - 1) \frac{E(dC)}{C} + \frac{1}{2} \frac{E(dC^2)}{C} \]  
(B.8)

If we assume that consumption and wealth grow at the same rate in equilibrium,

\[ \frac{E(dC)}{C} = \left( r \omega_k - r \omega_z + \eta f(\omega_k + \theta \omega^*_k) - \frac{C}{W} \right) dt, \quad \frac{E(dC^2)}{C} = \sigma_w^2 dt \]  
(B.9)

Plugging (B.9) into (B.8),

\[ \frac{E(dX_w)}{X_w} = (\gamma - 1) \left( r \omega_k - r \omega_z + \eta f(\omega_k + \theta \omega^*_k) - \frac{C}{W} \right) + \frac{1}{2} (\gamma - 1)(\gamma - 2) \sigma_w^2 \]  
(B.10)

Substitute (B.10) into (B.7), and use \( W = \widetilde{W}, \omega_k = \tilde{\omega}_k, \omega_z = \tilde{\omega}_z \) and \( \sigma_w^2 = \sigma_{w\sigma}^2 = \sigma_{w\sigma}^2 \), we obtain

\[ \frac{\tilde{C}}{W} = \frac{1}{1 - \gamma} \left( \beta - (r \tilde{\omega}_k - r \tilde{\omega}_z) - \gamma f(\omega_k + \theta \omega^*_k) \right) + \frac{\gamma}{2} \sigma_w^2 \]  
(B.11)
References


Economy,” *Journal of Economic Dynamics and Control*, 23 (5 – 6), April, 873 – 908.


Economies: Some Numerical Results and Empirical Evidence,” *Journal of International


